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THERMISTOR CALIBRATION PROCEDURE FOR SIMULATED SKIN SENSORS

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A thermistor calibration procedure is developed for silica-filled, alpha cellulose urea formaldehyde simulated skin sensors being used in the new Advanced Thermal Response Data Aquisition and Analysis System being built at the U. S. Army Natick Research, Development and Engineering Center. A thermistor parameter measurement procedure is devised from the results of an uncertainty analysis. The calibration procedure and uncertainty analysis are described in detail and results to date are presented, which show thermistor calibration precision within the predicted uncertainty and required system accuracy.

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PREFACE

This report summarizes the procedure for calibrating thermistors as the temperature sensing elements in simulated skin sensors. The sensors will be used in the new *Advanced Thermal Response Data Acquisition and Analysis System* (ATRES/DAAS). This work is being done under the project "Thermal Protection, 1L162786AH98CABOO", administered by the Physics & Engineering Branch (PEBr), Fiber and Polymer Science Division (FPSD), Soldier Science Directorate, U.S. Army Natick Research, Development, and Engineering Center.

All experiments are conducted at the Physics & Engineering Branch at the U.S. Army Natick Research, Development, and Engineering Center (Natick). The citation of trade names in this report does not constitute official endorsement or approval of use of an item.

THERMISTOR CALIBRATION PROCEDURE FOR SIMULATED SKIN SENSORS

INTRODUCTION

A new, state-of-the-art data acquisition/instrumented manikin system is being developed by the Physics and Engineering Branch at Natick to study the degree of soldier protection afforded by various uniforms and materials against thermal insults such as flames, CO_2 lasers, and thermonuclear weapons[1,2]. The *Advanced Thermal Response Data Acquisition and Analysis System* (ATRES/DAAS) consists of four units: 1) remote data collection sites, 2) master data collection control, conversion, and formatting, 3) data processing and storage, and 4) graphical analysis and presentation. This report deals with the remote data collection sites, more specifically, the temperature sensing elements that are mounted in system manikins.

Previous data acquisition/instrumented manikin systems used thermocouples as the temperature-sensing elements. The new ATRES/DAAS system uses thermistors. The previous thermocouple-mounted sensors were problematic in the low signal-to-noise ratio, which is characteristic of thermal electromotive forces (emfs). The high voltage output (volts) that can be realized with thermistors virtually eliminates this problem.

The thermistors, as with the previous thermocouples, are mounted inside simulated skin sensors. These sensors are disks, 2.5 cm in diameter and 1.25 cm thick, made from a 60/40 mixture (by weight) of alpha-cellulose-filled urea formaldehyde and silica. The disks have thermal and optical properties similar to those of human skin[3]. Skin burn severity can be predicted from the temperature recorded over time by the ATRES/DAAS system.

This report describes the development, implementation, and results of the thermistor calibration procedure currently being employed. First an uncertainty analysis is performed in order to determine appropriate parametric measurement procedures needed to obtain the required accuracy (± 0.5°C). The calibration procedure, based on the analysis, is then described in detail. Finally, results to date are presented, which show temperature measurement precision within the predicted uncertainty and required accuracy.

THEORETICAL BACKGROUND

Resistance thermometry is a temperature measurement technique that uses the change in electrical resistance of a material as its temperature changes. The two basic classes of resistance thermometers are resistance temperature detectors (RTD) and thermistors: the former employs a conductor and the latter, a semiconductor.

The Advanced Thermal Response Data Acquisition and Analysis System being developed at Natick uses negative temperature coefficient (NTC) thermistors as the temperature-sensing elements of the simulated skin sensors.

Thermistor Type

Thermistors were purchased from Thermometrics[4] as B05PA103N. The numbers and letters have the following significance:

B - Bead style: denotes a glass-coated bead.
05 - Bead diameter: 0.005 inch nominal diameter.
P - Lead configuration: denotes opposite leads.
A - Material system: determines the optimal range of resistance sensitivity for a particular temperature range.
103 - Zero power resistance: resistance at 25°C, 10,000 ohms.
N - Tolerance: denotes +- 25% resistance tolerance at 25°C.

B05 Thermal and Electrical Properties

Thermal time constant:	in still air - 0.12 s
	water plunge - 0.005 s
Dissipation constant:	in still air - 0.045mW/°C
	water plunge - 0.23mW/°C
Resistance range:	1K to 10M ohms
Maximum power rating:	0.006 watts.

Thermistors have two advantages over thermocouples. They have a higher signal-to-noise ratio ($\sim 10^3$), and, because of their smaller size, they respond much faster to temperature change. However, thermistors are fragile, have highly nonlinear resistance/temperature characteristics, are not interchangeable, and are expensive when close tolerance is required.

Linear Approximation

Although the resistance/temperature characteristics of thermistors are highly nonlinear, a linear approximation is obtained when the natural log of thermistor resistance is plotted versus the reciprocal of absolute temperature. In this case we have the relation

$$\ln R_T = A + \frac{\beta}{T} \tag{1}$$

where T is the absolute temperature (Kelvin) and β is a parameter dependent upon the material. Here, the slope β is considered constant although this approximation holds only over a relatively small temperature interval. For temperatures between 0°C and 50°C, interpolation errors introduced by assuming a constant β are on the order of 0.01°C over a 10°C temperature span, 0.04°C over a 20°C span, 0.1°C over a span of 30°C, and 0.3°C over a 50°C span.

The thermistor resistance at a reference temperature is given by the expression[5]

$$\ln R_{T_o} = A + \frac{\beta}{T_o} \tag{2}$$

where R_{To} is the resistance at reference temperature T_o. By subtracting Eq. 2 from Eq. 1 the standard thermistor equation is obtained[5]

$$\ln R_T - \ln R_{T_o} = \frac{\ln R_T}{\ln R_{T_o}} = \beta\left(\frac{1}{T} - \frac{1}{T_o}\right). \tag{3}$$

The value of β is dependent upon material, temperature, and thermistor construction and is considered unique for each thermistor. With two temperature/resistance data points, β can be directly determined using Eq. 3.

Polynomial Approximation

A more accurate approach uses a standard curve-fitting technique with the assumption that the ln R_T is a polynomial in 1/T, or vice versa[6]

$$\ln R_T = A_o + \frac{A_1}{T} + \frac{A_2}{T^2} + \ldots + \frac{A_N}{T^N}, \tag{4}$$

$$\frac{1}{T} = a_o + a_1(\ln R_T) + a_2(\ln R_T)^2 + \ldots + a_N(\ln R_T)^N. \tag{5}$$

These expressions give accurate representation of temperature/resistance characteristics over wide temperature spans. Thermometrics, Inc. reports an excellent curve fit using a third-degree polynomial with the squared term eliminated[5]. Equations 4 and 5 become

$$\ln R_T = A_o + \frac{A_1}{T} + \frac{A_3}{T^3} \tag{6}$$

and

$$\frac{1}{T} = a_o + a_1(\ln R_T) + a_3(\ln R_T)^3. \tag{7}$$

Eliminating the squared term reduces the number of points needed to determine polynomial constants but introduces some error. For example, for the temperature span $0<T<150°C$ the maximum error is 0.045°C.

As previously stated, β is the slope of the ln R_T versus 1/T (absolute) curve

$$\beta = \frac{d(\ln R_T)}{d(1/T)}. \tag{8}$$

Using Eq. 6, the expression for β is

$$\beta = A_1 + 3\frac{A_3}{T^2}. \tag{9}$$

Linear Voltage Divider

Thermistor resistance/temperature measurements are generally obtained through voltage measurements. The voltage divider is the simplest circuit used in thermistor networks (Fig. 1). In this case the output voltage $e_0(T)$ is measured across the fixed resistor R

$$e_o(T) = e_s\left(\frac{R}{R+R_T}\right). \tag{10}$$

Solving for R_T we obtain

$$R_T = R\left(\frac{e_s}{e_o(T)} - 1\right). \tag{11}$$

Advantages to using this circuit include increasing output voltage with increasing temperature and the ability to include the loading of any external measurement circuitry in the value of R. The value of R is chosen such that there is maximum linearity over the temperature range of interest[7].

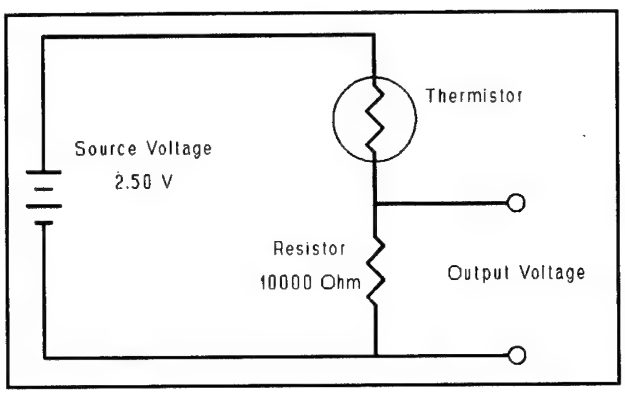

Fig 1. Voltage divider circuit for thermistor resistance measurements.

Thermal Time Constant Considerations

The thermal time constant of a thermistor in air is on the order of 0.12 seconds. This, combined with the extreme thermistor fragility, makes it extremely difficult to calibrate a thermistor as-is. That is, in order to

obtain R_{To}, the thermistor must be held at the constant reference temperature long enough for a resistance measurement to be obtained. With a 0.12 s thermal time constant, the thermistor reacts quickly to subtle temperature changes due to air currents, etc. Therefore, the thermistors are first mounted inside the simulated skin sensor. This insulates the thermistors from slight temperature changes and protects them from breakage due to handling.

With the thermistors mounted inside the sensors, the time constant of interest is that of the sensor material covering the thermistor. The time t required for heat to travel a distance X in a material with thermal diffusivity K can be approximated by the expression

$$t=\frac{x^2}{K}. \tag{12}$$

The thermal diffusivity K is defined as

$$K=\frac{k}{\rho c_p} \tag{13}$$

where k is the thermal conductivity, ρ is the material density and c_p is the specific heat at constant pressure. For a 60/40 mixture of urea-formaldehyde and silica, the values of k and ρc_p are reported to be[8]

k = 0.00131 cal/cm-s-°C,

ρc_p = 0.65 cal/cm^3-°C.

These values give a thermal diffusivity value of K = 0.0020 cm^2/s.

The sensors are manufactured with an approximately 0.05 cm thick of urea-formaldehyde and silica. The time constant calculated for this material thickness using Eq. 12 is about 1.25 seconds. The time for the applied heat to raise the temperature of the entire disk is also of interest. This will be important in calibrating the thermistors because it is important to have some idea how long it will take for a sensor to reach thermal equilibrium. For x=0.65 cm, which is about half the thickness of the sensor, the thermal time constant is computed to be about 5 min.

This relatively long thermal time constant indicates the importance of

allowing the skin sensor assembly to reach an equilibrium temperature before attempting to take temperature/resistance readings.

UNCERTAINTY ANALYSIS

In developing a thermistor calibration procedure it is advantageous to perform an uncertainty analysis in order to determine the most accurate measurement methods by which to quantify the parameters involved. Here the analysis is based on the standard root-sum-squares method.

If it is assumed that the uncertainties (W_1, W_2 ,...,W_N) in the independent variables (X_1, X_2,...,X_N) are all given with the same probability, then the uncertainty (W_R) in the dependent variable (R) is given by the expression

$$W_R=\left[\left(\frac{(\partial R)}{\partial X_1}W_1\right)^2+\left(\frac{(\partial R)}{\partial X_2}W_2\right)^2+\ldots+\left(\frac{(\partial R)}{\partial X_N}W_N\right)^2\right]^{1/2}. \tag{14}$$

We are interested in the accuracy with which we can measure both absolute and relative (ΔT) temperature.

Absolute-Temperature Uncertainty

The uncertainty analysis is based on the linear approximation. Using Eq. 3 and solving for T, expressions can be obtained for absolute-temperature as a function of thermistor resistance:

$$T=\frac{\beta T_o}{T_o \ln(R_T/R_{T_o})+\beta}. \tag{15}$$

For the above expression, there is expected uncertainty in β, R_T, T_o, and R_{To}. By inserting Eq. 15 into Eq. 14 a probable estimate of the uncertainty in T is obtained:

$$W_T=\left[\left(\frac{\partial T}{\partial R_T}W_{R_T}\right)^2+\left(\frac{\partial T}{\partial R_{T_o}}W_{R_{T_o}}\right)^2+\left(\frac{\partial T}{\partial \beta}W_\beta\right)^2+\left(\frac{\partial T}{\partial T_o}W_{T_o}\right)^2\right]^{1/2}, \tag{16}$$

where

$$\frac{\partial T}{\partial R_T}=\frac{-\beta\, T_o^2}{R_T(T_o\ln(R_T/R_{T_o})+\beta)^2}, \tag{17}$$

$$\frac{\partial T}{\partial R_{T_o}}=\frac{\beta\, T_o^2}{R_{T_o}(T_o\ln(R_T/R_{T_o})+\beta)^2}, \tag{18}$$

$$\frac{\partial T}{\partial T_o}=\frac{\beta^2}{(T_o\ln(R_T/R_{T_o})+\beta)^2}, \tag{19}$$

$$\frac{\partial T}{\partial \beta}=\frac{T_o^2\ln(R_T/R_{T_o})}{(T_o\ln(R_T/R_{T_o})+\beta)^2}. \tag{20}$$

The uncertainty in R_T is obtained by applying Eq. 11 to Eq. 14 with expected uncertainties in R, e_s, and $e_o(T)$:

$$WR_T=\left[\left(\frac{\partial R_T}{\partial R}W_R\right)^2+\left(\frac{\partial R_T}{\partial e_s}W_{e_s}\right)^2+\left(\frac{\partial R_T}{\partial e_o(T)}W_{e_o(T)}\right)^2\right]^{1/2}, \tag{21}$$

where

$$\frac{\partial R_T}{\partial R}=\left(\frac{e_s}{e_o(T)}-1\right), \tag{22}$$

$$\frac{\partial R_T}{\partial e_s}=\frac{R}{e_o(T)}, \tag{23}$$

$$\frac{\partial R_T}{\partial e_o(T)}=\frac{R\, e_s}{(e_o(T))^2}. \tag{24}$$

Using Eqs. 16-24 the uncertainty in R_T and the resulting uncertainty in T was calculated for R_{To} tolerances of 25%, 10%, 5%, and 1%, while varying the tolerance on R for values of 5%, 1%, and 0.1%. For this study the β value is obtained from the manufacturer[9] as well as the expected deviation[10]. It is assumed that the source-voltage is set using a digital voltmeter and the output voltage is recorded using LABTECH© NOTEBOOK. Values and tolerances for β, e_s, $e_o(T)$, and T_o are given as

β = 3395 K[9],
W_β = 101.85 K[10],
e_s = 2.5 Volts,
W_{es} = 0.01 Volts (readability of digital voltmeter),
$e_o(T)$ = 1.32 Volts @ 44°C (calculated using Eqs. 3 and 10),
$W_{eo(T)}$ = 1.32×10^{-4} Volts[11],
T_o = 25°C,
W_{To} = 0 (tolerance unknown, assumed negligible).

The resulting values are given in Table 1. Also included in Table 1 are the uncertainties associated with the individual terms. These are helpful in determining the major contributors to the overall uncertainty of R_T and T. These terms are designated as U and are determined by multiplying the parameter sensitivity by its respective tolerance as follows:

$$U_R = \left(\frac{e_s}{e_o(T)} - 1\right) W_R, \tag{25}$$

$$U_{e_s} = \frac{R}{e_o(T)} W_{e_s}, \tag{26}$$

$$U_{e_o(T)} = \frac{R\, e_s}{[e_o(T)]^2} W_{e_o(T)}, \tag{27}$$

$$U_{R_T} = \frac{\beta\, T_o^2}{R_T[T_o \ln(R_T/R_{T_o}) + \beta]^2} W_{R_T}, \tag{28}$$

$$U_{R_{T_o}} = \frac{\beta\, T_o^2}{R_{T_o}[T_o \ln(R_T/R_{T_o}) + \beta]^2} W_{R_{T_o}}, \tag{29}$$

$$U_\beta = \frac{T_o^2 \ln(R_T/R_{T_o})}{[T_o \ln(R_T/R_{T_o}) + \beta]^2} W_\beta. \tag{30}$$

Table 1. Uncertainty in R_T and absolute T with variation in R_{To} and R tolerances.

Tolerances		Uncertainty in R_T			Uncertainty in T (abs)				
R_{To}	R	U_R	U_{es}	U_{eoT}	W_{RT}	U_{RT}	U_{RTo}	U_B	W_T
25%	5%	1394	152	3.8	1402	8.2	7.4	0.6	11
10%	5%	1394	152	3.8	1402	8.2	3.0	0.6	8.7
5%	5%	1394	152	3.8	1402	8.2	1.5	0.6	8.4
1%	5%	1394	152	3.8	1402	8.2	0.3	0.6	8.2
25%	1%	279	152	3.8	317	1.8	7.4	0.6	7.6
10%	1%	279	152	3.8	317	1.8	3.0	0.6	3.5
5%	1%	279	152	3.8	317	1.8	1.5	0.6	2.4
1%	1%	279	152	3.8	317	1.8	0.3	0.6	2.0
25%	0.1%	28	152	3.8	155	0.9	7.4	0.6	7.5
10%	0.1%	28	152	3.8	155	0.9	3.0	0.6	3.1
5%	0.1%	28	152	3.8	155	0.9	1.5	0.6	1.8
1%	0.1%	28	152	3.8	155	0.9	0.3	0.6	1.1
A	B	C	D	E	F	G	H	I	J

As would be expected, the overall uncertainty in R_T (W_{RT}, column F) and T (W_T, column J) decreases as the R_{To} and R tolerances become smaller. From this table it is evident that the R tolerance is critical with as much as an 86% decrease in temperature uncertainty as the resistance tolerance changes from 5% to 0.1% (@ R_{To} tolerance of 1%). The required accuracy in absolute temperature measurement is ±0.5°C. Note that even with an R_{To} tolerance of 1% and an R tolerance 0.1%, the predicted uncertainty of 1.1°C is outside system requirements.

The 25% tolerance on reference-temperature resistance is unacceptable. It is therefore necessary to determine experimentally an R_{To} value for each thermistor. This implies that the assumption of zero uncertainty in the value of T_o (Eq. 16) is no longer valid.

Relative-Temperature Uncertainty

Because of the highly nonlinear thermistor-resistance/temperature characteristics, it is expected that there will be uncertainty in ΔT measurements as well as absolute-temperature measurements. Beginning with the linear approximation (Eq. 3), an expression can be obtained for ΔT

measurements which can then be applied to Eq. 14 to obtain expressions for relative-temperature uncertainty.

The change from some initial temperature T_i to some later temperature T_n can be written as

$$T = T_n - T_i . \tag{31}$$

Solving Eq. 3 for T and combining with Eq. 31 we obtain

$$\Delta T = \frac{\beta\, T_o}{(T_o \ln(R_{T_n}/R_{T_o}) + \beta)} - \frac{\beta\, T_o}{(T_0 \ln(R_{T_i}/R_{T_o}) + \beta)} . \tag{32}$$

Grouping like terms, and with a common denominator, we produce the expression

$$\Delta T = \beta\, T_o^2 \left[\frac{\ln(R_{T_i}/R_{T_o}) - \ln(R_{T_n}/R_{T_o})}{(T_o \ln(R_{T_n}/R_{T_o}) + \beta)\,(T_o \ln(R_{T_i}/R_{T_o}) + \beta)} \right. . \tag{33}$$

Applying the ln property; ln(a)-ln(b)=ln(a/b) to Eq. 33 the expression is

$$\Delta T = \frac{\beta\, T_o^2 \ln(R_{T_i}/R_{T_n})}{(T_o \ln(R_{T_N}/R_{T_o}) + \beta)\,(T_o \ln(R_{T_i}/R_{T_o}) + \beta)} . \tag{34}$$

We are interested in determining the uncertainty in the ΔT measurements. In Eq. 34 there is uncertainty in R_{To}, R_{Ti}, R_{Tn}, β, and T_o. Applying Eq. 34 to Eq. 14, the uncertainty in ΔT is given as

$$W_{\Delta T} = \left[\left(\frac{\partial \Delta T}{\partial \beta} W_\beta \right)^2 + \left(\frac{\partial \Delta T}{\partial T_o} W_{T_o} \right)^2 + \left(\frac{\partial \Delta T}{\partial R_{T_o}} W_{R_{T_o}} \right)^2 + \left(\frac{\partial \Delta T}{\partial R_{T_i}} W_{R_{T_i}} \right)^2 + \left(\frac{\partial \Delta T}{\partial R_{T_n}} W_{R_{T_n}} \right)^2 \right]^{1/2} , \tag{35}$$

where

$$\frac{\partial \Delta T}{\partial \beta} = \frac{T_o^2 \ln \dfrac{R_{T_i}}{R_{T_n}} \left[\left(T_o \ln \dfrac{R_{T_n}}{R_{T_o}} + \beta \right) \left(T_o \ln \dfrac{R_{T_i}}{R_{T_o}} + \beta \right) - \beta \left(2\beta + T_o \ln \dfrac{R_{T_i} R_{T_n}}{{R_{T_o}}^2} \right) \right]}{\left[\left(T_o \ln \dfrac{R_{T_n}}{R_{T_o}} + \beta \right) \left(T_o \ln \dfrac{R_{T_i}}{R_{T_o}} + \beta \right) \right]^2} , \tag{36}$$

$$\frac{\partial \Delta T}{\partial T_o} = \frac{\beta T_o \ln \dfrac{R_{T_i}}{R_{T_n}} \left\{ \left[2 \left(T_o \ln \dfrac{R_{T_n}}{R_{T_o}} + \beta \right) \left(T_o \ln \dfrac{R_{T_i}}{R_{T_o}} + \beta \right) \right] - \left[T_o \left(2 T_o \ln \dfrac{R_{T_n}}{R_{T_o}} \ln \dfrac{R_{T_i}}{R_{T_o}} + \beta \ln \dfrac{R_{T_i} R_{T_n}}{{R_{T_o}}^2} \right) \right] \right\}}{\left[\left(T_o \ln \dfrac{R_{T_n}}{R_{T_o}} + \beta \right) \left(T_o \ln \dfrac{R_{T_i}}{R_{T_o}} + \beta \right) \right]^2} , \tag{37}$$

$$\frac{\partial\Delta T}{\partial R_{T_o}}=\frac{\left(\dfrac{\beta T_o^3}{R_{T_o}}\ln\dfrac{R_{T_i}}{R_{T_n}}\right)\left(2\beta+T_o\ln\dfrac{R_{T_n}R_{T_i}}{R_{T_o}^2}\right)}{\left[\left(T_o\ln\dfrac{R_{T_n}}{R_{T_o}}+\beta\right)\left(T_o\ln\dfrac{R_{T_i}}{R_{T_o}}+\beta\right)\right]^2}, \tag{38}$$

$$\frac{\partial\Delta T}{\partial R_{T_i}}=\frac{\beta T_o^2\left\{\left[\left(T_o\ln\dfrac{R_{T_n}}{R_{T_o}}+\beta\right)\left(T_o\ln\dfrac{R_{T_i}}{R_{T_o}}+\beta\right)\right]-\left[T_o\ln\dfrac{R_{T_i}}{R_{T_n}}\left(T_o\ln\dfrac{R_{T_n}}{R_{T_o}}+\beta\right)\right]\right\}}{R_{T_i}\left[\left(T_o\ln\dfrac{R_{T_n}}{R_{T_o}}+\beta\right)\left(T_o\ln\dfrac{R_{T_i}}{R_{T_o}}+\beta\right)\right]^2}, \tag{39}$$

$$\frac{\partial\Delta T}{\partial R_{T_n}}=\frac{\beta T_o^2\left\{\left[\left(T_o\ln\dfrac{R_{T_n}}{R_{T_o}}+\beta\right)\left(T_o\ln\dfrac{R_{T_i}}{R_{T_o}}+\beta\right)\right]+\left[T_o\ln\dfrac{R_{T_i}}{R_{T_n}}\left(T_o\ln\dfrac{R_{T_i}}{R_{T_o}}+\beta\right)\right]\right\}}{R_{T_n}\left[\left(T_o\ln\dfrac{R_{T_n}}{R_{T_o}}+\beta\right)\left(T_o\ln\dfrac{R_{T_i}}{R_{T_o}}+\beta\right)\right]^2}. \tag{40}$$

Program ERROR.BAS

Equations 16-24 for absolute-temperature uncertainty and Eqs 35-40 for relative-temperature uncertainty were incorporated into a computer program written in BASIC entitled ERROR.BAS. The program gives temperature uncertainty in the initial absolute temperature T_i (presumably averaged over some number of points to minimize random noise effects), as well as the uncertainty in T_n, the absolute-temperature at some later time, and the predicted uncertainty in the difference of the two (ΔT). An example of the results of an analysis is given in Table 2. Input into the program includes the following:

β = 3474 Kelvin,
W_{β} = 104 Kelvin,
T_o = 25°C,
W_{To} = 0.01°C (assumed),
R_{Tr} = 10 kΩ,
W_{RTo} varies from 1% to 25%,
e_s = 2.5 volts,
W_{es} = 0.01 volts,
$e_o(T)$ varies with temperature,
$W_{eo(T)}$ = 0.01%,
R = 10 kΩ,
W_R = 0 (assumed that a close tolerance resistor is available).

Additional input is given in Table 2.

Table 2. Uncertainty in absolute and relative temperature with variation in reference temperature resistance tolerance W_{RTo}

Temp Input		Absolute Temp			Relative Temperature					
T_i	T_n	W_{RTo}	W_{Ti}	W_{Tn}	$W_{\Delta T}$	U_{β}	U_{To}	U_{RTo}	U_{RTi}	U_{RTn}
44	80	25%	7.44	9.44	2.35	1.0	0.0003	1.8	0.35	1.03
		10%	3.0	4.28	1.70	1.1	0.0003	0.71	0.35	1.03
		5%	1.64	2.87	1.58	1.1	0.0003	0.35	0.35	1.03
		1%	0.76	2.23	1.54	1.1	0.0003	0.07	0.35	1.03
34	44	25%	6.8	7.3	0.68	0.27	0.00003	0.45	0.26	0.35
		10%	2.7	3.0	0.54	0.27	0.00003	0.18	0.26	0.35
		5%	1.4	1.6	0.52	0.27	0.00003	0.09	0.26	0.35
		1%	0.47	0.76	0.51	0.27	0.00003	0.02	0.26	0.35
A	B	C	D	E	F	G	H	I	J	K

The U-terms in Table 2 (columns G-K) are similar to those defined in Eqs 23-28. They are the individual terms in the relative-temperature-uncertainty equation (Eq. 35) included here in order to determine the major sources of uncertainty in the final temperature uncertainty (columns D-F). Two temperature spans are considered in Table 2 in order to observe the effects on the W_{T} values (column F). As expected, the uncertainty in relative temperature increases with increasing temperature interval. For each case considered, the relative-temperature-uncertainty cannot be considered negligible. Note also that the error in β (column G) becomes comparatively substantial as the W_{RTo} tolerance decreases (for example in rows 4 and 8).

Thermistor Calibration Techniques for Improved Accuracy

From Tables 1 and 2 it is evident that the 25% R_{To} tolerance is unacceptable and will have to be improved. In order to obtain T_o and R_{To} values, accurate means of measuring temperature and resistance must be determined. A number of different schemes are considered here, including thermometer temperature measurements, temperature measurements made with a precalibrated thermistor, resistance measurements using a digital ohmmeter, and resistance values obtained through voltage measurements.

Uncertainty in R_{To} Measurements

In order to obtain R_{To} at reference-temperature T_o the sensor assembly must be brought to that reference temperature and the resistance recorded. Furthermore, in considering the sensor thermal time constant it was determined that readings should be taken at thermal equilibrium. It is the practical problem of bringing the sensor assembly to, say, 25°C ±0.1°C which makes this direct approach prohibitive.

An alternative approach is to take a reading at room temperature to obtain T and R_T, then designate a value for T_o and use Eq. 15 to solve for R_{To}.

$$R_{T_o}=\frac{R_T}{\exp\beta\left(\frac{1}{T}-\frac{1}{T_o}\right)} \tag{41}$$

If the temperature T is close to T_o (for example T=22°C and T_o= 25°C) the effects of nonlinear β on R_{To} will be minimal. However, with this indirect approach a degree of uncertainty is inherent in the resulting R_{To} due to uncertainties in the independent variables in Eq. 41. There will be uncertainty in T, β, and R_T. In order to quantify the overall uncertainty in the calculated R_{To} values, an uncertainty analysis is performed. Again, the analysis is based on the linear approximation. Applying Eq. 41 to Eq. 14 the uncertainty equation is obtained:

$$W_{R_{T_o}}=\left[\left(\frac{\partial R_{T_o}}{\partial R_T}W_{R_T}\right)^2+\left(\frac{\partial R_{T_o}}{\partial \beta}W_\beta\right)^2+\left(\frac{\partial R_{T_o}}{\partial T}W_T\right)^2\right]^{1/2}, \tag{42}$$

where

$$\frac{\partial R_{T_o}}{\partial R_T}=\left\{\exp\left[\beta\left(\frac{1}{T}-\frac{1}{T_o}\right)\right]\right\}^{-1}, \tag{43}$$

$$\frac{\partial R_{T_o}}{\partial \beta}=\frac{-R_T\left(\frac{1}{T}-\frac{1}{T_o}\right)}{\exp\beta\left(\frac{1}{T}-\frac{1}{T_o}\right)}, \tag{44}$$

$$\frac{\partial R_{T_o}}{\partial T}=\frac{R_T\beta}{T^2\left\{\exp\left[\beta\left(\frac{1}{T}-\frac{1}{T_o}\right)\right]\right\}}. \tag{45}$$

Equations 42-45 were incorporated into a computer program entitled RTNOT.BAS. The program was run for the case where resistance measurements are made using a digital voltmeter and temperature measurements are obtained with a National Bureau of Standards (NBS) calibrated thermometer with declinations of 0.1 C. Input into the program includes:

T = 22°C (ambient),
W_T = 0.1°C
R_T = 11,257 Ω (calculated with R_{To}=10,000 Ω),
W_{RT} = 0.1% of reading + 1 digit = 20 Ω or 0.2%[12],
B = 3474 K,
W_B = 104 K.

RTNOT.BAS gives an W_{RTo} value of 44 Ω or 0.4% of R_{To}--a substantial improvement over the manufacturer's tolerance of 2500 Ω (25% of 10,000 Ω). The individual U-terms, defined as the product of sensitivity Eqs. 43-45 and their respective tolerances, give U_{RT}=18, U_B=35, and U_T=20. The uncertainty in β is now the leading contributor to uncertainty in R_{To}.

The thermistor sensitivity in the temperature range of interest (35°C) is about 235 Ω/°C. With a 20 Ω tolerance on the ohmmeter used to obtain the resistance measurements, it is now possible to read resistance variations relating to temperature changes of 0.1°C. The 44 Ω value computed for the

case above relates to a temperature uncertainty of about 0.2°C. As a check, ERROR.BAS is run for the case where

$$T_o = T_i = T_n = 25°C$$
$$R_T = R = 10,000\ \Omega$$
$$e_s = 2.5 \text{ volts}$$
$$B = 3474 \text{ K}$$
$$W_B = 104 \text{ K}$$
$$W_{RTo} = 44\ \Omega$$
$$W_R = 10\ \Omega \text{ (0.1\% accuracy assumed)}$$
$$W_{es} = 0.01 \text{ volts}$$
$$W_{To} = 0.1°C.$$

The program gives a 0.25°C uncertainty in T_i and T_n.

Thermistor Calibration Using a Precalibrated Thermistor

The matching of thermal time constants between temperature standard and thermistor/sensor is an important consideration in thermistor calibration. To this end a procedure is considered which utilizes a precalibrated thermistor for temperature measurements.

Initially, one or a number of thermistor/sensors are calibrated as described in the previous section using an ohmmeter for resistance measurements and a NBS calibrated thermometer for temperature measurements. Once the R_{To} values have been determined for the reference thermistor/sensors, they can be used to measure the temperature T relating to the resistance values R_T from which R_{To} can be calculated for the remaining thermistor/sensor assemblies. Presumably with this approach, experimental error incurred due to mismatched time constants will be reduced. A computer program was written to calibrate thermistor/sensors with a precalibrated thermistor/sensor. The program RTNOTA.BAS was obtained by modifying RTNOT.BAS (see Appendix). It uses the reference-thermistor/sensor R_T value to calculate the temperature, which is assumed to be identical for both the reference- and test-thermistor/sensor assemblies (kept in close proximity during testing). This calculated temperature is then used to calculate the R_{To} value of the thermistor being calibrated.

The program uses Eqs. 42-45 to calculate the uncertainty in the R_{To} measurements so that for each run the expected degree of accuracy can be monitored. It differs from program RTNOT.BAS in that it uses a temperature uncertainty (W_T) of 0.25°C (as determined in the previous section) instead of a W_T of 0.1°C associated with direct readings from the NBS calibrated thermometer. A major concern with this approach is that the 0.25°C W_T value is too great an uncertainty to be introduced at this level of the calibration procedure.

An analysis was performed which uses ERROR.BAS to determine the uncertainty in temperature measurements made during calibration with an NBS calibrated thermometer as compared to those made with a precalibrated thermistor/sensor assembly. The results are documented in Table 3.

Table 3. Final temperature uncertainty for calibration temperature measurements obtained with a thermometer compared to those made with a thermistor.

	Thermometer			Thermistor		
	A	B	C	D	E	F
Input						
T_o	22	25	25	22	25	25
T_i	25	25	25	25	25	25
T_n	25	44	80	25	44	80
R	10,000	10,000	10,000	10,000	10,000	10,000
RT_o	11,258	10,000	10,000	11,258	10,000	10,000
e_s	2.5	2.5	2.5	2.5	2.5	2.5
W_B	104	104	104	104	104	104
W_{RTo}	20	40	40	20	40	40
W_R	10	10	10	10	10	10
W_{es}	0.01	0.01	0.01	0.01	0.01	0.01
W_T	0.05	0.24	0.24	0.24	0.34	0.34
Output						
W_{Ti}	0.24	0.34	0.33	0.34	0.41	0.41
W_{Tn}	0.24	0.76	2.23	0.34	0.81	2.26
$W_{\Delta T}$	0.29	0.64	1.91	0.30	0.64	1.91
U-Terms						
U_B	0	0.5	1.6	0	0.5	1.6
U_{To}	0	0.001	0.009	0	0.001	0.01
U_{RTo}	0	0.01	0.04	0	0.01	0.04
U_{RTi}	0.21	0.21	0.21	0.21	0.21	0.21
U_{RTn}	0.21	0.35	1.1	0.21	0.35	1.1

Note: Beta held constant throughout at 3474 K, and R_{To} input values in columns A and D were calculated using the linear approximation.

Columns A, B, and C are for the case of thermometer temperature measurements and columns D, E, and F are for thermistor temperature measurements. Column A depicts the case where the R_{To} value (@ 25°C) of a thermistor is being determined from resistance/temperature data obtained at room temperature (22°C). The input temperature uncertainty (W_T) in column A is 0.05°C. (Here it is assumed that the 0.1°C declinations on the thermometer can be read within ±0.05°C.) The W_{RTo} value of 20 Ω assumes resistance measurements made with a digital ohmmeter and the W_R value of 10 Ω assumes a 0.1% resistor tolerance. The output shows an absolute-temperature uncertainty of 0.24°C. This value is used as input for column B. It means that the uncertainty in the T_o value of 25°C is 0.24°C. Note also that the reference temperature resistance tolerance (W_{RTo}) has increased from 20 Ω in column A to 40 Ω in column B; this, too, is due to solving for the thermistor R_{To} value

from data taken at 22°C rather than obtaining the value directly at 25°C.

The output for column B shows an increasing temperature uncertainty from that of column A. Note that the W_{Tn} value relates to the input value of 44°C. The only difference between columns B and C is the increased temperature span to 80°C. In all three columns the relative-temperature uncertainty is substantial. The U-Terms show β to be the major contributor to final temperature measurement uncertainty.

In examining the values in columns D, E, and F, which relate to the case of thermistor temperature calibration with a precalibrated thermistor, it can be seen that the overall temperature uncertainty shown in the output is greater than in columns A, B, and C. In column D the temperature uncertainty input (W_T) is given as 0.24°C. This, as in column A, relates to the case where the R_{To} (@ 25°C) value of the thermistor is being determined indirectly from temperature/resistance data obtained at ambient temperature (22°C). Because the thermistor depicted in column D is being calibrated with a precalibrated thermistor/sensor assembly, it is exactly the same case as that depicted in column B with the exception that the W_{RTo} value is 20 Ω (measurement with ohmmeter). The thermistor has not yet acquired the uncertainty which is introduced by indirectly obtaining R_{To} values from data obtained at 22°C. It is not until column E that this uncertainty is introduced.

Again, the only difference between the input values in columns E and F is the T_n values of 44°C and 80°C, respectively. The output temperature errors shown in columns B, C, E, and F are simply too great to introduce during thermistor calibration. The use of a precalibrated thermistor for thermistor calibration is not practical with the temperature and resistance measuring techniques considered here. The U-term values given in columns E and F show β as the major source of final temperature uncertainty.

Experimental Determination of Beta for Improved Accuracy

It can be seen in Tables 3 and 4 that the greatest error incurred in temperature measurements is due to uncertainty in β. Thermometrics[10] reports that β varies with a production lot by 3% and from lot to lot by up to 5%. We have been assuming a 3% variation. At this point this represents the major limit in our ability to accurately calibrate thermistor/sensor assemblies. It has been reported that β should be considered unique to each thermistor and should be experimentally determined[13]. Whereas it is necessary to experimentally determine R_{T_0} through resistance/temperature measurements at one or a number of points, it is possible, at the same time, to use this data experimentally to determine β.

In order to establish the accuracy with which β can be found experimentally, an uncertainty analysis is performed based on the linear approximation. Solving Eq. 3 for β we obtain

$$\beta = \frac{T\ T_o}{(T_o - T)} \ln \frac{R_T}{R_{T_o}} . \tag{46}$$

Uncertainty in experimentally determined β is expected as a result of uncertainty in the independent variables R_T, R_{T_0}, T and T_o. The uncertainty equation (Eq. 14) becomes

$$W_\beta = \left[\left(\frac{\partial \beta}{\partial R_T} W_{R_T} \right)^2 + \left(\frac{\partial \beta}{\partial R_{T_o}} W_{R_{T_o}} \right)^2 + \left(\frac{\partial \beta}{\partial T} W_T \right)^2 + \left(\frac{\partial \beta}{\partial T_o} W_{T_o} \right)^2 \right]^{1/2} , \tag{47}$$

where

$$\frac{\partial \beta}{\partial R_T} = \frac{T\,T_o}{R_T (T_o - T)} , \tag{48}$$

$$\frac{\partial \beta}{\partial R_{T_o}} = \frac{-T\,T_o}{R_{T_o} (T_o - T)} , \tag{49}$$

$$\frac{\partial\beta}{\partial T}=\frac{T_o^2}{(T_o-T)^2}\ln\frac{R_T}{R_{T_o}}, \qquad (50)$$

$$\frac{\partial\beta}{\partial T_o}=\frac{-T^2}{(T_o-T)^2}\ln\frac{R_T}{R_{T_o}}. \qquad (51)$$

These expressions were incorporated into a computer program ER_BETA.BAS, and a study was performed using this program to determine experimental β uncertainty. The results are given in Table 4.

From Eq. 46 it is evident that in order to calculate β using the linear approximation, two temperature/resistance data points must be experimentally obtained. If β is temperature dependent, then it is important to determine the temperature range over which we are concerned. It has been concluded that this temperature range is between 22°C and 50°C. Determining β, the slope of the $\ln(R_T)$ versus 1/T plot, between 22°C and 50°C will give us our most accurate temperature measurements at a point halfway between those two points, or in the 36°C vicinity. Similarly, determining β between 36 and 50°C we determine the slope in the 43°C region and obtain our most accurate readings in that region. With this in mind, β uncertainty was determined for a number of cases. In columns A and B of Table 4 the reference-temperature is taken to be 36°C with an additional point at 22°C. Note also the difference in temperature measurement uncertainty in the input variables W_{TO} and W_T. In column A it is assumed that the NBS calibrated thermometer can be read to ±0.1°C. In column B a ±0.05°C readability is assumed. In the former case the U-Terms show temperature measurement uncertainty is the primary source of β uncertainty. In the latter case it is our ability to measure resistance accurately (again, it is assumed that resistance measurements are obtained with a digital ohmmeter).

Table 4. Uncertainty in experimentally determined β.

	A	B	C	D	E	F
Input						
T_o	36	36	22	36	50	36
T	22	22	36	50	36	22
W_{To}	0.1	0.05	0.05	0.05	0.05	0.05
W_T	0.1	0.05	0.05	0.05	0.05	0.05
W_{RTo}	20	20	20	20	20	2
W_{RT}	20	20	20	20	20	2
Output						
W_B	42	29	29	45	45	17.7
$\%W_B$	1.2	0.8	0.8	1.3	1.3	0.51
U-Terms						
U_{RT}	12	12	20	35	22	1.2
U_{RTo}	20	20	12	22	35	2.0
U_T	26	13	12	12	13	13
U_{To}	24	12	13	13	12	12

Note: Beta held constant throughout at 3474 K.

Alternating reference temperatures has no effect on β uncertainty as can be seen in the output W_B values in columns B and C. Similarly, in columns D and E temperatures of 36°C and 50°C are alternated as reference temperature with the same result. Beta uncertainty does increase at elevated temperature intervals as seen by the increase in W_B. Output $\%W_B$ terms should be compared to the manufacturer's β tolerance of 3%. In column F, resistance measurement uncertainty is decreased in order to determine the effect on our ability to measure β. The improvement is substantial with a 60% decrease in β uncertainty, and a 95% decrease in U-Terms; U_{RT} and U_{RTo}. Overall system accuracy is greatly improved with more accurate resistance measurements.

The question now arises as to how this increased beta precision affects the overall temperature measurement uncertainty. A study was performed using program ERROR.BAS for this purpose. The program was modified to include in the output the U-Terms from the absolute-uncertainty expressions as well as those from the relative-temperature uncertainty expressions. The results of this study are documented in Table 5.

Table 5. Effects of increased β accuracy on temperature measurement uncertainty.

	A	B	C	D
Input				
T_o	36	36	36	36
T_i	22	30	22	30
T_n	50	45	50	45
R_{To}	6603	6603	6603	6603
e_s	2.5	2.5	2.5	2.5
W_B	35	35	17	17
W_{RTo}	20	20	2	2
W_{To}	0.05	0.05	0.05	0.05
Output				
W_{Ti}	0.25	0.26	0.20	0.24
W_{Tn}	0.45	0.39	0.43	0.37
$W_{\Delta T}$	0.50	0.44	0.47	0.43
U-Terms for $W_{\Delta T}$				
U_B	0.2	0.1	0.1	0.05
U_{To}	0.00003	0.00004	0.00003	0.00004
U_{RTo}	0.015	0.008	0.0015	0.0008
U_{RTi}	0.19	0.23	0.19	0.23
U_{RTn}	0.42	0.36	0.42	0.35
U-Terms for W_{Ti}				
U_B	0.13	0.06	0.07	0.03
U_{To}	0.05	0.05	0.05	0.05
U_{RTo}	0.08	0.08	0.008	0.008
U_{RTi}	0.19	0.23	0.19	0.23
U-Terms for W_{Tn}				
U	0.15	0.09	0.07	0.05
U_{To}	0.05	0.05	0.05	0.05
U_{RTo}	0.09	0.09	0.009	0.009
U_{RTn}	0.41	0.36	0.42	0.36

Note: Input R_{To} calculated using linear approximation. Beta held constant throughout at 3474 K.

In columns A and B it is assumed that resistance is measured with an ohmmeter. Columns C and D assume a more accurate resistance measurement capability, which manifests itself in lower input W_B and W_{RTo} values. The temperature interval for columns A and C is 22°C to 50°C. For columns B and D it is 30°C to 45°C. In each case the reference temperature is 36°C. Temperature measurements are assumed to be made with a thermometer. The higher β accuracy is evidenced by a decrease in absolute and relative temperature uncertainty now shown to be 0.50°C and below. U-terms show the primary source of temperature measurement uncertainty to be in the resistance measurements rather than the β uncertainty.

If, in calibrating the thermistor/sensor assemblies, temperature/resistance measurements are obtained at three temperatures, Eq. 46 can be solved for 3 different cases:

1. B_{LM}: T_o, R_{To} @ 36°C
   T, R_T @ 22°C
2. B_{MH}: T_o, R_{To} @ 36°C
   T, R_T @ 50°C
3. B_{LH}: T_o, R_{To} @ 22°C
   T, R_T @ 50°C.

Where the subscripts L, M, and H denote low, medium, and high temperatures, respectively. Additionally, a $\beta_{average}$ value can be determined using the expression

$$\beta_{average}=\frac{(\beta_{LM}+\beta_{MH}+\beta_{LH})}{3}. \qquad (52)$$

This value is expected to be very close to the value of B_{LH}.

A computer program was developed to calculate these four β values. The program, EX_BETA.BAS, also uses Eqs. 47-51 to determine the uncertainty in each β value. This ability has been included in the program so that variations in the four β values can be compared to the predicted uncertainty in each value. If β value variations are consistently outside the predicted uncertainties, this can be taken as evidence of a temperature-dependent β which, if a high degree of accuracy is needed, can be accounted for by some polynomial β expression.

Determination of Thermistor Resistance Using Voltage Measurements

It has been shown (Table 5) that the primary source of temperature uncertainty is in the resistance measurements when the temperature/resistance values are obtained with a NBS-calibrated thermometer and digital ohmmeter. The ±20 Ω meter accuracy causes uncertainty in both the R_{To} and β values.

Another approach is considered which utilizes voltage measurements obtained using Labtech Notebook. Resistance values are then computed using Eq. 11:

$$R_T = R\left(\frac{e_s}{e_o(T)} - 1\right). \tag{11}$$

In addition, the source voltage can be recorded simultaneously. Source-voltage drift is a problem that, up to now, has not been addressed. Using Eqs. 21-24 the uncertainty in the R_T measurements for this case was predicted. Using the values

e_s = 2.57 volts (obtained experimentally),
W_{es} = 2.57x10^{-4} volts (0.01% tolerance),
$e_o(T)$ = 1.32 volts (obtained experimentally),
$W_{eo(T)}$ = 1.32x10^{-4} volts (0.01% tolerance),
R = 9,998.8 Ω,
W_R = 0.2 Ω (1% tolerance resistor calibrated at Natick RD & E electronics lab to a tolerance of 0.002%).

Eq. 14 gives a W_{RT} value of 2.8 Ω (0.03 % tolerance); a substantial improvement over the 20 Ω tolerance obtained with ohmmeter resistance measurements. When the new R_T tolerance is taken into account, the final temperature measurement accuracy is greatly increased. From ERROR.BAS we obtain the following numbers:

Table 6. Temperature uncertainties for case of resistance values obtained through voltage measurements.

	A	B
Input		
β	3474	3474
T_o	36	36
T_i	25	25
T_n	50	80
R_{To}	6603	6603
e_s	2.57	2.57
W_B	18	18
W_{RTo}	2.8	2.8
W_{To}	0.05	0.05
Output		
W_{Ti}	0.07	0.07
W_{Tn}	0.09	0.27
$W_{\Delta T}$	0.096	0.23

Two temperature ranges are considered in Table 6; in column A the

temperature span is 25°C - 50°C with the maximum output uncertainty (0.096°C) appearing in the relative-temperature term, while in column B the temperature span is 25°C - 80°C with the maximum output uncertainty (0.27°C) appearing in the high-temperature term. The input W_{To} value of 0.05°C assumes temperature measurements made with a thermometer. The U-terms (not included) show the primary source of error to be in β due to temperature measurements of ±0.05°C accuracy.

It is important to remember that the uncertainty analysis is based upon the constant β linear approximation. For the 25°C - 80°C temperature range in column B, the actual error in an 80°C measurement could be much greater (±0.4°C) than that shown in W_{Tn} due to variations in β. For our temperature range (20°C - 50°C), however, β variations are expected to introduce an additional error of ± 0.1°C. When this is added to the output in column A we are still inside accuracy requirements, and the determination of thermistor resistance through voltage measurements represents an acceptable thermistor calibration approach.

RTNOT.BAS Output for Voltage R_T Measurements

In the section entitled Uncertainty in R_{To} Measurements, a procedure was investigated whereby the R_{To} value of a thermistor is calculated using the linear approximation from temperature/resistance values recorded at a temperature close to, but not equal to, T_o. It was found that this procedure introduces additional uncertainty into the R_{To} value. RTNOT.BAS gave a W_{RTo} value of 44 ohms for this case.

With more accurate resistance values obtained through voltage measurements, this procedure might now be feasible. An experimental run is performed where temperature/resistance data are taken at 22.20°C. Using these data R_{To} is calculated for T_o = 25°C. RTNOT.BAS is run again with the following results:

Input
T (ambient) = 22.20°C
W_T = 0.05°C (thermometer)
β = 3474 K (approximation)
W_B = 18 K
R_T (ambient) = 11,415.5 ohms (experimental)
W_{RT} (ambient) = 3 ohms.

Output
R_{To} = 10,222 Ω @ 25°C
Tolerance = 2.22% (relative to R_{To} = 10,000 Ω, as purchased)
W_{RTo} = 21.3 Ω
W_{RTo}% = 0.21% (relative to R_{To} = 10,222 Ω)

U-Terms
U_{RT} = 2.7
U_B = 5.9
U_T = 20

The output now shows a W_{RTo} value of 21.3 Ω, a 100% improvement over the previously determined 44 Ω case, but far from the 3 Ω uncertainty obtained when the R_{To} value is acquired at the exact temperature to which it relates.

Temperature Correction Terms

Temperature measurement error is introduced in the form of a bias when power dissipated in a thermistor causes it to self-heat. This section describes the formulation of a corrective term, which eliminates this bias. In addition, an expression is presented for an emergent stem correction which is required when a thermometer is used as the calibration standard.

Experimental Investigation of Thermistor Self-Heating

Power is dissipated in a thermistor in the form of heat when connected to an electrical circuit. This causes the thermistor temperature to rise above the ambient, introducing an error between the actual ambient temperature and the temperature of the thermistor. An energy balance can be written as

$$\frac{dQ_{in}}{dt}=\frac{dQ_{lost}}{dt}+\frac{dQ_{absorbed}}{dt}. \tag{53}$$

This states that the rate at which energy is supplied to a thermistor is equal

to the rate at which it is lost to its surroundings plus the rate at which it is absorbed. The rate at which energy is supplied is equal to the power dissipated in the thermistor

$$\frac{dQ_{\epsilon}}{dt} = P = [e_o(T)]\,I = I^2 R_T = \frac{[e_o(T)]^2}{R_T}, \tag{54}$$

where $e_o(T)$ is the voltage across, and R_T, the resistance in the thermistor. The rate at which heat is lost is proportional to the thermistor temperature rise:

$$\frac{dQ_{lost}}{dt} = \delta\,(T - T_a)\,. \tag{55}$$

Where δ is the dissipation constant, T is the thermistor temperature, and T_a is the ambient temperature. Power is absorbed by a thermistor according to the expression

$$\frac{dQ_{abs}}{dt} = c_p m \frac{dT}{dt}, \tag{56}$$

where c_p is the specific heat of the thermistor material and m is its mass. Combining Eqs. 53-56 we obtain the expression

$$\frac{dQ_{\epsilon}}{dt} = \frac{[e_o(T)]^2}{R_T} = \delta\,(T - T_a) + c_p m \frac{dT}{dt}\,. \tag{57}$$

After a relatively short period of time (t > thermal time constant = 0.21 s), the thermistor is at thermal equilibrium and the absorption term drops out of Eq. 57 leaving the expression:

$$P = \delta\,(T - T_a)\,. \tag{58}$$

In order to quantify the thermistor temperature rise due to self-heating, the dissipation constant δ must be determined. If two power readings are taken we obtain two expressions

$$P_1 = \delta\,(T_1 - T_a) \tag{59}$$

and

$$P_2=\delta(T_2-T_a)\,. \tag{60}$$

In order to eliminate T_a, Eq. 60 is subtracted from Eq. 59, which is valid providing the ambient temperature does not change. The expression becomes

$$(P_1-P_2)=\delta(T_1-T_2) \tag{61}$$

or

$$\delta=\frac{P_1-P_2}{T_1-T_2}\,. \tag{62}$$

Power terms can be calculated using the relation

$$P=\frac{[e_o(T)]^2}{R_T}, \tag{63}$$

where, for a linear voltage divider

$$e_o(T)=e_s\left(\frac{R_T}{R_T+R}\right) \tag{64}$$

where e_s is the source voltage and R is the resistance of the voltage divider resistor. If the δ value is now inserted into Eq. 58, the temperature rise corresponding to a given power can be computed:

$$\frac{P}{\delta}=T_{SH}=(T-T_a)\,. \tag{65}$$

Table 7 gives the results of an experimental test run to determine δ for thermistor/sensor assembly T_{58}.

Table 7. Experimental determination of dissipation constant δ at 21.85°C.

Temp	Run	e_S	e_T	R_T	T	P	P/δ	T - P/δ
21.85	1	1.094	0.589	11700	21.59	2.96E-5	0.087	21.50
	2	2.257	1.211	11588	21.86	1.27E-4	0.375	21.49
	3	3.781	2.005	11308	22,54	3.56E-4	1.052	21.50
	4	4.327	2.281	11175	22.88	4.66E-4	1.380	21.50

The temperature shown in the first column is that of the thermometer and

was held constant for each of the four runs. In order to ensure a constant T_a, the system was brought to thermal equilibrium and source-voltage was turned on only while data were being acquired. The source-voltage e_s was varied and the voltage across the thermistor was calculated for each case using Eq. 64. The variation in R_T values in Table 7 shows the error introduced by power dissipation. This is also evidenced in the rising temperatures, which were calculated using the linear approximation and previously determined β and R_{To} values. Power was calculated using Eq. 63. The P/δ entries are from Eq. 65 and have units of degrees Celsius. These are the corrective values. When the corrective term is subtracted from the calculated temperature, the final temperature is approximately the same for each run. These corrected temperatures should be the same as the thermometer temperature (in this case they are not because the correction was not implemented in the original determination of β and R_{To}).

The value of the dissipation constant used above was determined by applying Eq. 62 first to the data obtained in runs 1 and 2, then to the data obtained in runs 1 and 3, etc. until all possible combinations were exhausted. The value of δ determined for each combination is shown in Table 8.

Table 8. Dissipation constant values determined using data given in Table 7.

	Combination	Dissipation Constant (W/°C)
Temp = 21.85°C	1-2	3.58×10^{-4}
	1-3	3.39×10^{-4}
	1-4	3.37×10^{-4}
	2-3	3.33×10^{-4}
	2-4	3.32×10^{-4}
	3-4	3.30×10^{-4}

Average value used in Table 7 = 3.38×10^{-4} W/°C
Standard deviation = 1.02×10^{-5} W/°C

The study shown in Tables 7 and 8 was repeated at a higher temperature to determine the degree to which the dissipation constant is temperature dependent. The results are given in Tables 9 and 10.

Table 9. Experimental determination of dissipation constant δ at 49.15°C.

Temp	Run	e_S	e_I	R_I	T	P	P/δ	$T - P/\delta$
49.15	1	0.821	0.260	4627.7	48.31	1.46E-5	0.041	48.27
	2	2.676	0.839	4563.3	48.74	1.54E-4	0.434	48.31
	3	4.549	1.400	4452.7	49.49	4.40E-4	1.239	48.25

Table 10. Dissipation constant values determined using data given in Table 9.

	Combination	Dissipation Constant W/°C
Temp = 49.15°C	1-2	3.24×10^{-4}
	1-3	3.60×10^{-4}
	2-3	3.80×10^{-4}

Average value used in Table 9 = 3.55×10^{-4} W/°C
Standard deviation = 2.84×10^{-5} W/°C

The relatively small (5%) change in δ from Table 8 to Table 10 is an indication that the value of δ is not greatly affected by changes in temperature (though it would require a larger number of samples to determine a statistically sound basis for that assumption). The amount of power dissipated in a thermistor/sensor, however, is strongly temperature dependent.

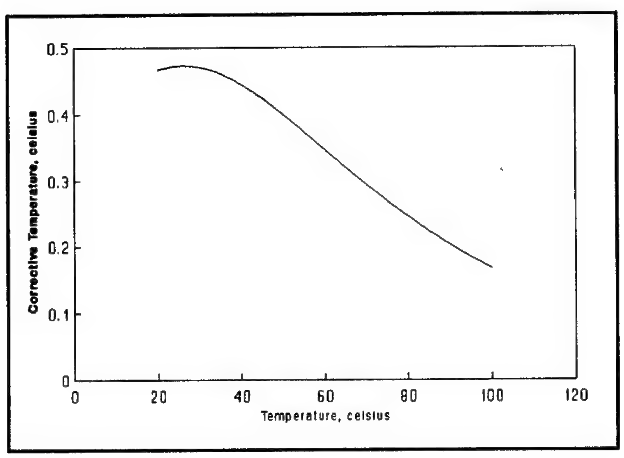

Fig. 2. Self-heating temperature correction curve for thermistor/sensor T_{58}.

Figure 2 is a plot of corrective temperature (P/δ) versus absolute-temperature for thermistor/sensor T_{58}. The plot shows the error incurred if self-heating is not taken into account. Upon subtraction of this curve, the data will relate to the desired ambient temperature.

Figure 2 was generated using the linear approximation (Eq. 3). Power dissipated was determined using Eqs. 63 and 64. The corrective-temperature term was calculated using Eq. 65. In addition, the following values were used:

B_{LM} = 3368 K (experimental),

R_{To} = 7003 ohms (experimental)

@ T_o = 36°C,

$\delta = 3.3 \times 10^{-4}$ Watts/°C,

e_s = 2.5 V.

The dissipation constant has been previously given as 0.045 mW/°C in still air and 0.23 mW/°C in the water plunge. The nominal value computed here of 0.33 mW/°C represents a decrease in temperature rise due to self-heating when the thermistors are mounted in sensors.

In Tables 8 and 10 standard deviations of 0.01 mW/°C and 0.028 mW/°C, respectively, are given. In order to determine the significance of these variations, an uncertainty analysis is performed. Combining Eq. 14 with Eq. 62, an expression is obtained for the uncertainty in δ:

$$W_\delta = \left[\left(\frac{\partial\delta}{\partial P_1} W_{P_1}\right)^2 + \left(\frac{\partial\delta}{\partial P_2} W_{P_2}\right)^2 + \left(\frac{\partial\delta}{\partial T_1} W_{T_1}\right)^2 + \left(\frac{\partial\delta}{\partial T_2} W_{T_2}\right)^2\right]^{1/2}, \tag{66}$$

where

$$\frac{\partial\delta}{\partial P_1} = (T_1 - T_2)^{-1}, \tag{67}$$

$$\frac{\partial\delta}{\partial P_2} = -(T_1 - T_2)^{-1}, \tag{68}$$

$$\frac{\partial\delta}{\partial T_1} = -\frac{(P_1 - P_2)}{(T_1 - T_2)^2}, \tag{69}$$

$$\frac{\partial\delta}{\partial T_2}=\frac{(P_1-P_2)}{(T_1-T_2)^2}. \tag{70}$$

The uncertainty in T_1 and T_2 is known to be approximately 0.1°C. The values of W_{P1} and W_{P2} can be obtained by combining Eq. 14 with Eq. 63:

$$W_P=\left[\left(\frac{\partial P}{\partial e_o(T)}W_{e_o(T)}\right)^2+\left(\frac{\partial P}{\partial R_T}W_{R_T}\right)^2\right]^{1/2}, \tag{71}$$

where

$$\frac{\partial P}{\partial e_o(T)}=\frac{2[e_o(T)]}{R_T} \tag{72}$$

and

$$\frac{\partial P}{\partial R_T}=-\frac{[e_o(T)]^2}{(R_T)^2}. \tag{73}$$

The uncertainty in thermistor voltage ($W_{eo(T)}$) is unknown but can be computed by combining Eq. 14 with Eq. 64. The expression is

$$W_{e_o(T)}=\left[\left(\frac{\partial e_o(T)}{\partial e_s}W_{e_s}\right)^2+\left(\frac{\partial e_o(T)}{\partial R_T}W_{R_T}\right)^2+\left(\frac{\partial e_o(T)}{\partial R}W_R\right)^2\right]^{1/2}, \tag{74}$$

where

$$\frac{\partial e_o(T)}{\partial e_s}=\frac{R_T}{(R_T+R)}, \tag{75}$$

$$\frac{\partial e_o(T)}{\partial R_T}=\frac{(e_s)R}{(R_T+R)^2}, \tag{76}$$

$$\frac{\partial e_o(T)}{\partial R}=-\frac{(e_s)R_T}{(R_T+R)^2}. \tag{77}$$

Using the experimental data of runs 1 and 4 in Table 7 and Eqs. 67-77, the uncertainty in δ is computed by Eq. 66 to be 0.023 mW/°C. This is quite close to the computed experimental standard deviations of 0.01 mW/°C and

0.028 mW/°C, and is taken as an indication that this is a reliable technique by which to determine the dissipation constant δ. In terms of temperature, this 0.023 mW/°C uncertainty in the dissipation constant corresponds to ±0.04°C at 25°C and a source-voltage of 2.5 V.

Because the dissipation constant is highly dependent on heat transfer mechanisms, which can vary from one thermistor to another, it is necessary to determine a unique dissipation constant for each thermistor/sensor. Program EX_BETA.BAS was modified to include the calculation of δ. The new program, DC_BETA.BAS, requires that an additional run be made at low temperature, which will provide the information necessary to solve Eqs. 62-65. This run is performed at a lower source-voltage (power) than that used to determine β and R_{To}.

As can be seen from Eq. 62, in order to calculate δ, the temperatures T_1 and T_2 relating to powers P_1 and P_2 must be calculated from temperature/resistance data obtained in the same run.

$$\delta = \frac{P_1 - P_2}{T_1 - T_2} \tag{62}$$

In order to determine these temperatures, β must be known for that particular thermistor/sensor. Beta cannot be accurately determined without the self-heating correction.

For this reason an iterative technique is implemented whereby a value of δ is assumed, self-heating is computed based on that assumption, temperature is corrected, β is determined, corrected temperatures are calculated, which are then used to determine a new δ, etc. The process continues until the difference between any two consecutive values of δ is less than $1\text{x}10^{-6}$ W/°C. This value is approximately an order of magnitude less than the predicted δ uncertainty.

With the value of δ known for each thermistor/sensor the self-heating correction can be implemented, depending on the required accuracy, either by assuming an average δ value for all thermistor/sensor assemblies, or by

allowing each to retain its own unique value. In either case, a single expression, obtained by combining Eqs 63-65, can be used to obtain the 'actual' ambient temperature;

$$T_{actual}=T_T-\left[\left(\frac{e_s}{R_T+R}\right)^2 \frac{R_T}{\delta}\right]. \tag{78}$$

The alternative to implementing corrections for thermistor self-heating is to operate at a source voltage for which thermistor self-heating can be considered negligible. From Fig. 2 it can be seen that the maximum error due to thermistor self-heating occurs at about 25°C. Fig. 3 shows a curve which was generated using the second term on the right of Eq. 78;

$$\frac{P}{\delta}=\left(\frac{e_s}{R_T+R}\right)^2 \frac{R_T}{\delta}. \tag{79}$$

An R_T value relating to a temperature of 25°C (10,000 Ω) was held constant. A value of 3.3×10^{-4} W/°C was used for the dissipation constant, an

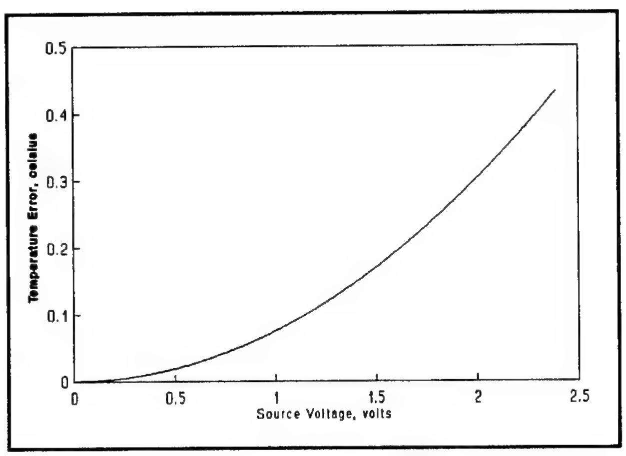

Fig 3. Temperature error versus source voltage for thermistor self-heating at 25°C and $\delta = 3.3\times10^{-4}$ W/°C.

R value of 10,000 Ω was used, and the source voltage was allowed to vary from 0 volts to 2.5 volts. The curve shows that a source voltage of 1.2 V or less

would keep the maximum error inside that of experimental uncertainty. For a source-voltage in the vicinity of 1 V, experimental data have shown thermistor voltages ranging from about 0.2 V to 1 V for the temperature range of interest.

Thermometer Emergent-Stem Correction

The NBS calibrated thermometer used as the calibration standard is a total-immersion thermometer. It is not, however, being used as such. A correction must be made to compensate for that portion of the thermometer exposed to room temperature when readings are obtained above room temperature. The commonly used equation for this correction is [14]

$$T_{sc}=K\,n\,(T-t)\,, \tag{80}$$

where K is the differential expansion coefficient of mercury, n is the number of degrees emergent from the bath, T is the temperature of the bath, and t is the mean temperature of the emergent-stem. Here, the values are

K = 0.00016,

n = 111°C,

T = 36°C and 50°C,

t : determined experimentally.

The value of t is determined by fastening another thermometer to the stem of the standard at approximately the halfway point. With t = 25°C the emergent-stem correction is 0.20°C @ 36°C and 0.44°C @ 50°C. Both of these values are outside our predicted temperature measurement uncertainty and, therefore, must be taken into consideration during thermistor/sensor calibration.

Summary of Uncertainty Analysis

It has been shown that in order to achieve the required temperature measurement precision of ±0.5°C, a thermistor calibration procedure must be implemented. Furthermore, it is possible to attain the required tolerances

with equipment that is currently available at Natick. The procedure requires the acquisition of four temperature/resistance data points at three different temperatures. The temperatures correspond to the midpoint and extremes of the temperature range of interest, which was designated to be 22°C - 50°C (midpoint at 36°C).

The dissipation constant is determined at 22°C by taking readings at two power settings relating to source-voltage inputs of 0.5 V and 2.5 V. Obtaining the dissipation constant at this temperature ensures the greatest degree of accuracy because of the high thermistor sensitivity to self-heating at this temperature (greatest change in dissipated power per unit change in source-voltage). The uncertainty analysis predicts variations in δ to be in the vicinity of a standard deviation of 0.025 mW/°C.

Reference-temperature resistance (R_{To}) relating to a reference temperature (T_0) of 36°C is calculated from temperature/voltage data taken close to, but, in general, not equal to, 36°C. It was determined in the analysis that the uncertainty in these values is on the order of 20 Ω.

The practice of relating all R_{To} values to the same T_0 value of 36°C is not necessary if, in the ATRES/DAAS software, all thermistor sensors are allowed to retain their unique constants in the linear approximation, or if a polynomial expression is used to relate temperature to resistance. In the linear approximation, greater accuracy is obtained when R_{To} relates to the midpoint temperature. The common reference-temperature is used here in order to observe statistical variations in thermistor constants as a check on the manufacturer's stated tolerances.

The determination of the slope, β, requires two data points. Four values of β are determined in the calibration process; first using the low temperature T_L and the midpoint temperature T_M from which we obtain β_{LM}, then using the midpoint temperature and high temperature T_H giving a slope value β_{MH} between these two points. Another β value (β_{LH}) is obtained using the low and high temperature end points, and finally an average slope (β_{ave}) is calculated using the three values.

Each of the β values (except β_{ave}) has a corresponding uncertainty which

is monitored throughout the calibration process. If it is assumed that a plot of the natural log of thermistor resistance versus the reciprocal of absolute temperature is a straight line over the 22°C to 50°C temperature range, then variations in β values should be within, or close to, the predicted uncertainties. Variations which fall outside predicted β uncertainties can be taken as an indication either of unreliable temperature/resistance data or that the thermistor curve cannot be considered linear over the designated temperature range. The maximum uncertainty in β occurs for β_{MH} and is generally in the vicinity of 23 Kelvin. The most accurate β value is β_{LH} with predicted uncertainties of about 8 Kelvin.

The development of the calibration procedure can be summed up in a series of plots, which are generated using an altered version of program ERROR.BAS. The plots show the uncertainty in T (thermistor temperature) resulting from variations of the measurement tolerance (%) for β, T_o, R, e_s, and $e_o(T)$. For each plot the following nominal values were used:

β = 3500 K

W_β = 20 K,

R_{To} = 7,000 Ω,

W_{RTo} = 25 Ω,

T_o = 36°C,

W_{To} = 0.1°C,

e_s = 2.50 V,

$W_{es} = W_{eo(Ti)} = W_{eo(Tn)}$ = 0.0001 V.

The tolerances listed above were held constant except where they were varied for a specific plot. The initial, later, and Delta temperature curves show temperature uncertainty at 25°C, 50°C, and the difference (ΔT) between the two, respectively. In each plot (Figs 4-8), the percent tolerance on one independent variable is allowed to vary from zero to the initial or precalibration tolerance (for example, thermistors were purchased with a manufacturer's stated β tolerance of 5% but are now being calibrated to a tolerance of 0.5%) while all other independent variables are held constant at the current or calibrated tolerance. This is summarized in Table 11.

Table 11. Thermistor parametric tolerances before and after refinement and calibration.

Variable	Initial or Precalibrated Tolerance	Current or Calibrated Tolerance	Plot
β	5%	0.5%	0%-5%
R_{To}	25%	0.25%	0%-25%
R	5%	0.01%	0%-5%
e_s	0.4%	0.004%	0%-0.4%
$e_o(T)$	0.4%	0.004%	0%-0.4%

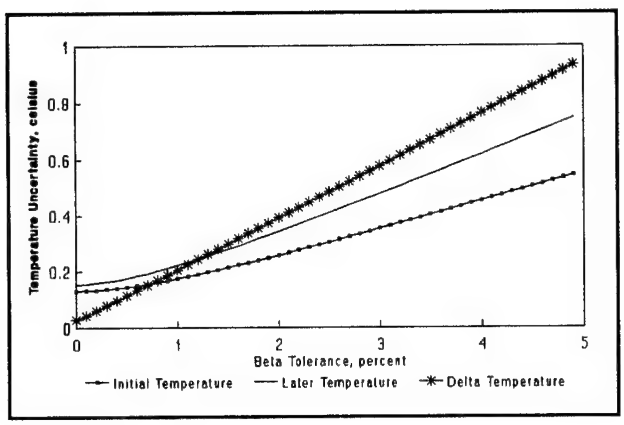

Fig. 4. Temperature uncertainty due to increasing β tolerance.

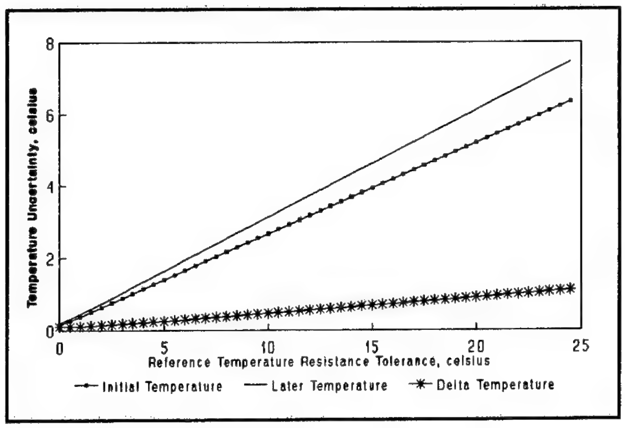

Fig. 5. Temperature uncertainty due to increasing reference temperature resistance (R_{T_0}) tolerance.

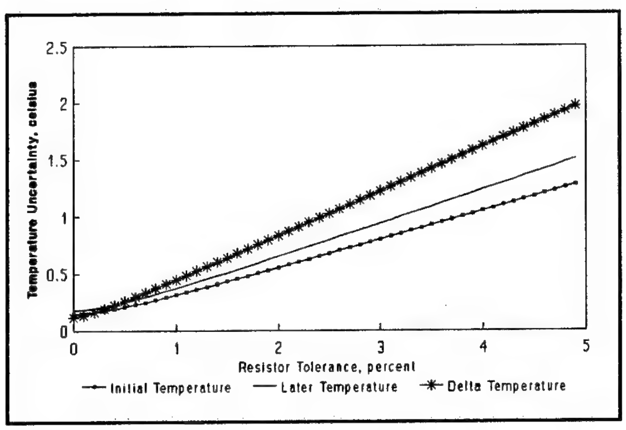

Fig. 6. Temperature uncertainty due to voltage divider impedance (R) tolerance.

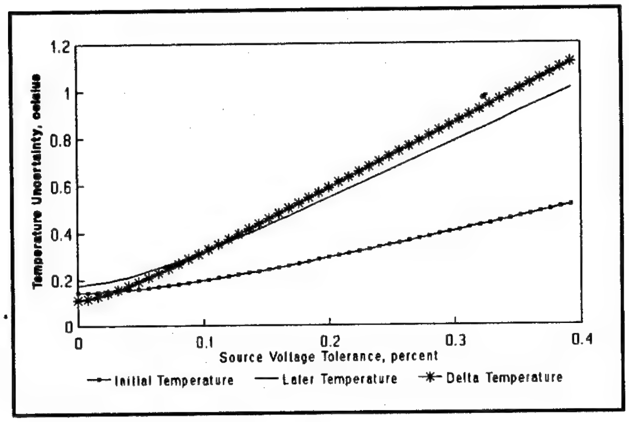

Fig. 7. Temperature uncertainty due to source-voltage (e_s) tolerance.

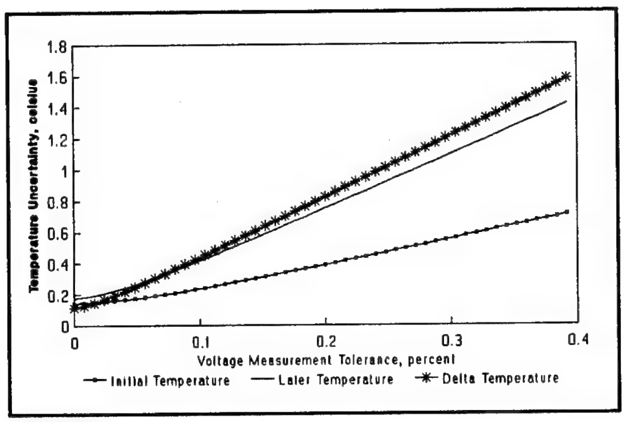

Fig. 8. Temperature uncertainty due to voltage measurement (e_s and $e_0(T)$) tolerance.

Figure 6 underscores the importance of having a precisely known, stable circuit impedance. Figure 7 was included to show error incurred when the source-voltage drifts and is not precisely monitored (25% variations have been noted in the calibration process). This plot differs from the next in that

Fig. 8 shows uncertainty due to the accuracy of all voltage measurements combined.

Note that no plot is given for temperature uncertainty versus thermistor resistance R_T. This is because we are not measuring resistance directly, but rather the voltage across the voltage divider resistor R. Any plot which varies a parameter which affects the accuracy of the R_T values (e_s, $e_o(T_i)$, $e_o(T_n)$ and R), shows, indirectly, the effect on R_T.

Additional Considerations

In addition to the error sources and corrective terms already presented, the effects of thermistor aging and instability, lead-wire resistance, direct-current error, and system noise should also be considered.

Thermistor aging is a well documented phenomenon[15,16,17]. While it is known that the problem is related to the bulk semiconductor material (nickel-manganese oxides), the exact mechanisms governing thermistor instability due to aging are unknown. Studies show a substantial increase in the stability of bead- and disk-type thermistors when they are encapsulated in glass but the reasons for this, too, are not fully understood.

A plot of temperature error due to aging versus log time will generally be a straight line for thermistors stored at constant temperature. That is, the same change in thermistor characteristics will be encountered in the period of 0 to 10 hours as in 10 to 100 hours. Table 12 shows the results of an experiment in which groups of 5 to 10 thermistors were aged--each group at a different constant temperature--for periods of 25 and 5000 hours[16]. Column A gives the mean temperature shift for each group. These should be compared to the predicted shifts in column B obtained by curve-fitting the experimental data to an equation of the form

$$y=x^{a}\,b, \tag{81}$$

where y is the temperature-shift in mK, x is the hours of aging, and the exponent a and factor b are functions of Kelvin temperature.

Table 12. Actual vs. predicted shifts due to thermistor-aging [16].

Age Temperature	Time Hours	A Actual Shift mK	B Predicted Shift mK	C Error A-B mK
-80	25	0	+8	-8
-80	5000	+2	+6	-4
25	25	+1	+9	-3
25	5000	+2	+7	-5
70	25	+1	+1	0
70	5000	0	+3	-3
100	25	-1	-2	+1
100	5000	-8	-5	-3
150	25	-7	-9	+2
150	5000	-25	-34	+9
200	25	-21	-19	-2
200	5000	-122	-106	-12
250	25	-32	-34	+2
250	5000	-252	-268	+16

The first several decades of temperature error can be eliminated by elevating the thermistor to its highest operating temperature. This condition is realized when the thermistors are molded into the sensor disk. In this step, the thermistor is brought from about 100°C to a maximum temperature of 150°C and back down in a process which takes about 1 hour.

Though the effects of aging instability as presented in Table 12 are clearly within our predicted temperature uncertainty, they can be conveniently monitored periodically at the triple-point of water. Because, with the use of an ice bath, thermometer measurements are not necessary, a higher degree of measurement precision can be obtained, thereby bringing the variations into our measurement realm. It is expected that changes in the output versus

temperature relationship will be apparent even outside the calibrated range[18,19].

The relatively high thermistor impedance renders leadwire resistance negligible[20]. For our system, leadwire resistance has been experimentally determined to be in the vicinity of 1 Ω--within the 3 Ω uncertainty and hence is not being compensated for.

When direct current is run through a thermistor, errors can occur due to ionic conduction in the thermistor and EMFs created at the junction of dissimilar metals in the circuit. Hajiev and Agarunov[21] describe polarization and electrolysis processes which can occur due to ionic conduction in thermistors. This can result in irreversible changes in thermistor characteristics. They report some thermistors which are initially unstable and others which become unstable after being subjected to direct current for a few days or in some instances a few hours. The effects of thermistor instability due to direct current can be eliminated by use of an alternating current bridge or by periodically reversing the current through the thermistor. Similarly, errors caused by thermal EMFs within a circuit can be minimized using the same techniques[22,23].

While instability errors due to direct-current ionic conduction have not been quantified, those due to thermal EMFs are expected to be relatively small (for example the EMF generated at a platinum-copper junction is typically 6-8 $\mu V/°C$) over our temperature range of interest, and within our predicted uncertainty. However, in cases where high precision is necessary the effects of direct-current errors should be investigated.

Because of the high output voltage associated with thermometry (1-2 V over 20°C - 50°C temperature range with 2.5 V source voltage), the effects of noise (electrical and quantization) are expected to be small. A study was conducted where the voltage noise and signal-to-noise ratio were determined for three thermistor/sensors at temperatures of 23°C and 50°C. It was found that the average standard deviation of the noise in a constant voltage (temperature) signal was 0.0018 V on a 1.124 V signal at 23°C and 0.0017 V on a 1.78 V signal at 50°C. These relate to signal/noise ratios of 600 and 1000,

respectively. After processing, the noise is in terms of temperature, and is experimentally verified to be in the vicinity of 0.05°C at room temperature for a signal/noise ratio of about 500. Steps can be taken to further reduce electrical noise by shielding cables, etc., and quantization noise can be reduced by signal amplification prior to A/D conversion, though it is not necessary at this time.

EXPERIMENTAL METHOD

Instrumentation

Experiments were conducted at the PEBr, FPSD, U.S. Army Natick Research, Development, and Engineering Center, Natick, MA. Figure 9 shows a sketch of the experimental setup, followed by a list and brief description of the equipment currently being used in the thermistor calibration procedure.

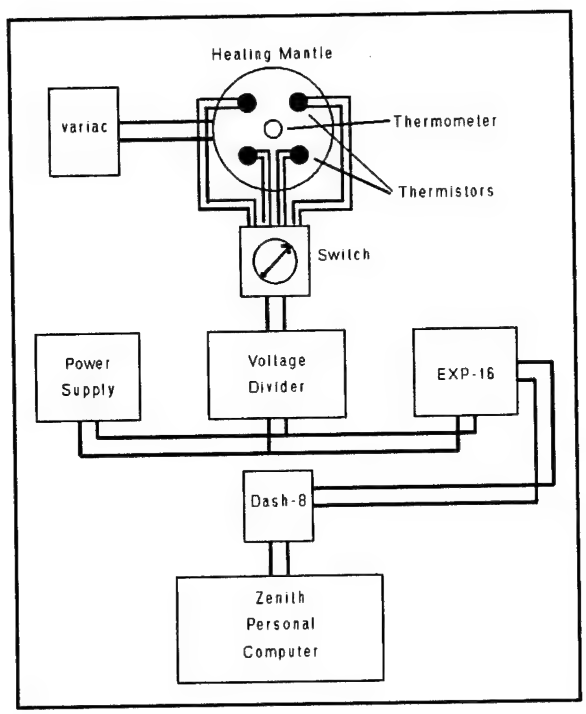

Fig. 9. Experimental setup for thermistor calibration.

Thermometer-1 - Mercury total immersion, nitrogen filled. Range from -10°C to 101°C in declinations of 0.1°C. Built by Arthur H. Thomas Co., Phil., PA. Calibrated by National Bureau of Standards. Thermometer-1 is used to monitor the temperature of the bath.

Thermometer-2 - Mercury total immersion. Range from -0.6°C to 100.4°C in declinations of 0.2°C, used to determine temperature of stem of thermometer-1 for emergent-stem correction.

Power supply - Lambda Electronics Corp., Melville, L.I., New York. Range of 0-20 VDC, used as thermistor power supply.

Multimeter - Fluke, Model 8024A Multimeter, John Fluke Mfg. Co., Inc., Seattle, Washington. Used to set voltage supply with ±0.02 V accuracy.

Heating Mantle - Glas-Col Apparatus Co., Terre Haute, Indiana, Cat. No. TM-108. Hemispherical well filled with 200-mesh copper powder.

Aluminum Holder - Built at Natick, disk diameter=3.125 inches, 1 inch thick. Drilled to hold four thermistor/sensor assemblies and mercury well of Thermometer-1.

Wood Cover - Built at Natick, disk diameter=6 inches, 1 inch thick. Drilled to allow access to thermistor/sensor assembly terminals. Mounts on top of heating mantle to reduce convective losses.

Variac Autotransformer - General Radio Company, Concord, MA, type W5MT3, load 0-140 V. Used to control power dissipation in heating mantle.

Switch - 12 position, used to select thermistor/sensor assembly under study.

Linear Voltage Divider - Built at Natick. Contains single 10 K resistor calibrated at Natick. True resistance at 24.8 C = 9998.8 Ω ±0.002%, 0±50 ppm/°C.

Hardware -

1. Zenith Data Systems Model ZWX-0248-62 PC - Heath Company, Benton Harbor, Michigan. Used in data acquisition and processing.

2. Dash-8 - Data Acquisition and Control Interface Board installed in Zenith PC to provide analog/digital interface and high-speed data acquisition.

3. EXP-16 - Expansion Multiplexer and Instrumentation Amplifier used to extend Dash-8 capabilities. Both Dash-8 and EXP-16 are products of MetraByte Corporation, Taunton, MA.

Software -

1. Labtech Notebook™ - Software package from Laboratory Technologies Corporation, Wilmington, MA. Used in data acquisition. Interfaces with Dash-8 and EXP-16.

2. Lotus 1-2-3™ - LOTUS, Cambridge, MA. Used to process data files generated with Labtech Notebook.

Experimental Procedure

1. Labtech Notebook is set up for two input channels; one to monitor source voltage and the other for voltage-divider output-voltage. Sampling rate is set at 50 Hz (normal speed) for a duration of 10 s. Both channels are dumped into one file, which is placed in the 1-2-3 directory.

2. Four thermistor/sensor assemblies are placed in the aluminum holder which is buried in the 200-mesh copper powder inside the well of the heating mantle. A cardboard cover is placed over the aluminum holder as an additional

insulator and the wooden cover is placed over the whole assembly such that the sensor terminals can be accessed through the drilled holes.

3. Four pairs of alligator clips are attached to the terminals taking special note that the clip numbers correspond to the proper location of the sensor in the Al holder. Plastic funnels are then fitted over the alligator clips and the foam plugs are inserted into the funnel openings to reduce convective losses.

4. The 12-position switch is set to a neutral position and the source-voltage is turned on and adjusted with the Multimeter to an output of 0.50 V. (Note: Allowing a current to run through a sensor will cause it to heat up and prevent the assembly from reaching thermal equilibrium.)

5. An O-ring is slid over the mercury well of thermometer-1 and positioned where the glass meets the mercury well. The O-ring should be positioned such that it rests on top of the wood insulator plate. This reduces convective losses through the accommodating hole and also prevents the mercury well from resting on the bottom of the Al holder. (Note: Allowing the thermometer bulb to contact the Al holder in this vertical position will create pressure in the bulb and a bias in the temperature reading.)

6. Thermometer-2 is attached to thermometer-1 such that the two are in contact along the length of the stem. The system is now allowed to sit for 20 - 30 min to allow the assembly to reach thermal equilibrium. It is important that the room temperature remains stable within this period because the stem of thermometer-1 is acting as a heat-transfer fin. Once the assembly reaches thermal equilibrium (evidenced by no change in temperature indicated by thermometer-1), the low-power room-temperature data can be taken.

7. The switch is positioned such that current is running through the sensor in location-1 of the Al holder. Acquisition of data is begun with

Labtech Notebook. The switch is left in position-1 for two seconds, then in position-2 for two seconds, etc., until data have been collected for all four sensors, after which the switch should again be set to a neutral position.

8. Labtech Notebook is exited and Lotus 1-2-3 is entered, retrieving file CAL1.WK1. The Labtech data file is imported into the appropriate spreadsheet column and the file is saved under a new name, making note of the calibration batch (for example, BATCH1.WK1).

9. Returning to Labtech Notebook, the source-voltage is changed to 2.50 V using the Multimeter to monitor the output.

10. It is important to observe whether the temperature has changed according to thermometer-1. (A 0.05°C change is acceptable, but a change of 0.1°C or greater is not, and the complete process from Step 6 must be repeated.) Step 7 must be repeated at the new voltage setting and the Labtech Notebook data must be entered into the appropriate column of the new Lotus file.

11. The temperature should be recorded in the Lotus spreadsheet and also on the Calibration Sheet. Lotus will automatically average the low- and high-power source-voltage (columns A and E). These must be entered into the Calibration Sheet. The voltage corresponding to each sensor must be averaged using Lotus, and these values entered in the Calibration Sheet for both low- and high-power.

12. The variac is turned on at a setting of 30. The heating mantle should be allowed to increase in temperature until thermometer-1 reads 28°C. At this point the variac is turned off and the assembly is allowed to reach thermal equilibrium. The system should equilibrate close to 36°C--the midpoint temperature.

13. Thermometer-1 must be observed closely, and the voltage data recorded as described above at the point where the assembly just reaches thermal equilibrium close to 36°C. The stem temperature must also be recorded as indicated by thermometer-2. (In general, thermometer readings are always taken as the temperature moves from low to high, because this is the way in which thermometers are calibrated both by the manufacturers and by the National Bureau of Standards.)

14. After completion of Step 13, the variac is again turned on and left on until thermometer-1 reads 43.5°C. The system will reach equilibrium close to 50°C. The necessary data are recorded as with the midpoint temperature. There is now the necessary information to calculate thermistor constants R_{To}, β, and δ.

15. Thermistor constants are calculated using the QuickBASIC[24] program entitled DC_BETA.BAS. When run, the program prompts for the following information:

TL - Low temperature obtained with thermometer-1.

VSL - Low-temperature high-power source-voltage.

LOW POWER VSL - Low-temperature low-power source-voltage.

TM - Midpoint-temperature obtained with thermometer-1.

VSM - Midpoint-temperature source-voltage.

STEM TEMP (M) - Stem-temperature at midpoint.

TH - High-temperature obtained with thermometer-1

VSH - High-temperature source-voltage.

STEM TEMP (H) - Stem-temperature at high point.

VTL - Voltage-divider voltage at low-temperature high-power.

LOW POWER VTL - Voltage-divider voltage at low-temperature low-power.

VTM - Voltage-divider voltage at midpoint-temp.

VTH - Voltage-divider voltage at high-temp.

The system iterates on the dissipation constant then gives the output as in the following example:

Table 13. Output of program DC_BETA.BAS for sensor T97.

	BETA, Kelvin	Uncertainty	RT, ohms
BLM	3496.13	20.08985	12525.42
BMH	3497.361	18.16944	7790.396
BLH	3496.759	9.629001	4742.007
BAVE	3496.75		
	TEMP	EXP TEMP	ERROR
LOW	24.02902	24.0306	-1.577377E-03
MED	36.59214		
HIGH	50.76709	50.7698	-2.708435E-03
RT36	7942.424		
DC	2.948801E-04		
SHL	SHM	SHH	
0.5290134	0.5266719	0.4669011	

Given under the heading 'BETA' are the four β values and in the next column labeled 'UNCERTAINTY' is the uncertainty in units of Kelvin for each of the respective β values. These should be compared to the 'BAVE' value. The three resistance values listed under 'RT' correspond to the three temperatures listed under 'TEMP'. The temperatures listed under 'TEMP' are the 'actual' thermistor temperatures obtained by adding the self-heating values listed under 'SHL', 'SHM', and 'SHH', and the emergent-stem corrections to the temperatures obtained with thermometer-1 during the calibration process. The temperatures given under 'EXP TEMP' are the temperatures calculated using the linear approximation, the listed RT values, and the experimentally determined β_{ave} and R_{To} values. The 'ERROR' values are obtained by subtracting the 'EXP TEMP' listings from the 'TEMP' listings and are included as a check--they have no statistical significance. The thermistor reference-temperature resistance R_{To} relating to T_0=36°C is given as 'RT36', and the dissipation constant is given as 'DC'. All of these values should be entered into the Thermistor Data Sheet. This completes the thermistor calibration procedure.

RESULTS

The uncertainty analysis predicts a standard deviation between 0.1°C and 0.2°C over the temperature range of 22°C to 50°C. Results to date show absolute variations of 0.1°C or less. At the writing of this report, 64 sensors have been calibrated as described above. In order to quantify the precision with which the thermistors have been calibrated, three different studies were performed. In each study, three or four sensors are compared.

The inputs range from 20°C to 80°C in steps of 0.1°C. Thermistor resistance is calculated over this temperature range for the group using each sensor's unique β, R_{To}, and δ value. The dissipation constant is implemented such that thermistor self-heating is subtracted from the temperatures from which the R_T values were calculated (2.50 V source-voltage is assumed). The three or four temperature curves are then averaged and each curve is subtracted from the average creating absolute-error curves which are plotted on a single graph for comparison.

In the first study, six sensors, chosen at random from different batches, are calibrated, three times each, to check for consistency in results. The curves from each of the three calibration runs are then plotted,

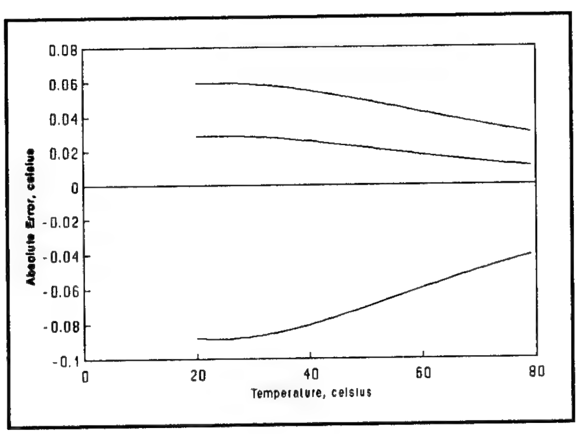

Fig. 10. Absolute-error curves for three calibration runs for sensor T76.

as described above, for each sensor. The maximum deviation from the average for any one sensor occured with sensor T76 and was about 0.085°C. This is shown in Fig 10.

The smallest deviation from the average occurred with sensor T80 and was about 0.028°C. This is shown in Fig. 11.

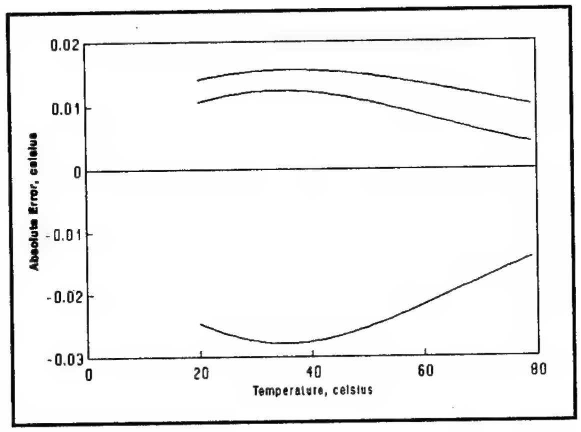

Fig. 11. Absolute-error curves for three calibration runs for sensor T80.

For the second study, 19 batches of four sensors each were plotted as described above in order to observe variations within a batch. For each of the 19 batches, absolute deviations are within 0.1°C from average. Figure 12 shows the error curves for Batch-19, which exhibited the greatest deviation from average.

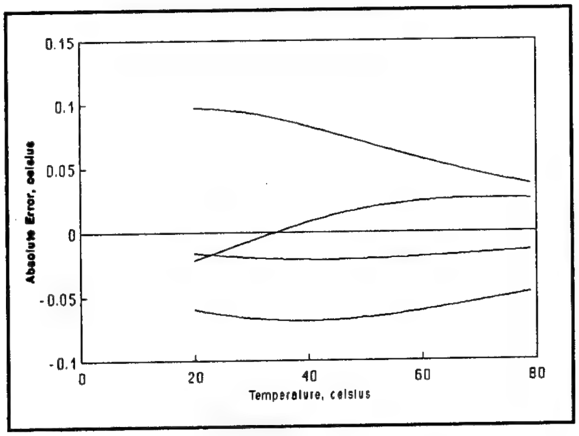

Fig. 12. Absolute-error curves for sensors in Batch-19.

An exceptionally precise batch, Batch-5A, is shown in Fig. 13.

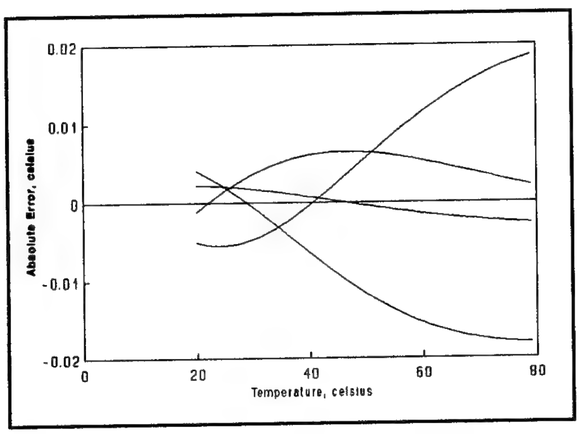

Fig. 13. Absolute-error curves for sensors in Batch-5A.

For the third study, individual sensors were randomly selected--one from each batch--and grouped into four batch-mixes. This was done in order to observe variations from one batch to another. For each batch-mix, absolute deviations are less than 0.1°C and within predicted uncertainty. An example is shown in Fig. 14.

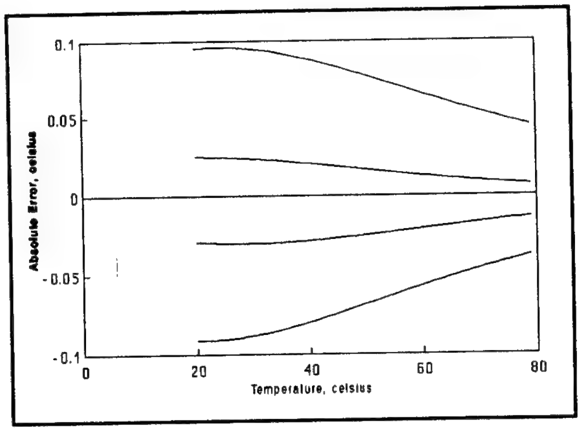

Fig. 14. Absolute-error curves for Batch-mix 3.

It should be noted that in the above three studies it is assumed that there is a random scatter of data, and that the average of a number of curves is the most accurate. The consistency of the results would seem to support this assumption. However, it is possible that the entire data base is biased, and the random variations are superimposed over this bias. Because we have no additional standard against which to verify our results (i.e., a factory-calibrated thermistor), there is no way of knowing. However, because the current standard is a NBS-calibrated thermometer, it is believed that if there is such a bias, it is probably quite small and presumably within predicted uncertainty. Though we cannot at the present time verify high accuracy, we have demonstrated high precision.

In developing the software to be used in data acquisition and processing, it is necessary to know the magnitude of variations in thermistor constants. Recall that the thermistors were purchased with an R_{To} tolerance of ±25% relative to 10,000 Ω at T_0=25°C. Also, β values were expected to vary, for a given batch, by ±3% from some nominal value, and for different batches, by as much as 5%. From the 64 thermistor/sensors calibrated to date, the following information was obtained:

Reference-Temperature Resistance R_{To} (36°C):

Average = 6736 Ω

Standard Deviation = 564 Ω (8%)

Maximum Value = 10213 Ω

Minimum Value = 6193 Ω

% Maximum Deviation > Average = 52%

% Maximum Deviation < Average = 8%

The large % maximum deviation for values greater than average is evidence of the existence of an outlier, verified to be the maximum value shown above. This unusually high R_{To} value was double-checked by calibrating the thermistor a second time, and was found to be correct. When the outlier is removed, however, the variations fall within the manufacturer's stated tolerance of ± 25% as shown below:

Average = 6681 Ω

Standard Deviation = 355 Ω (5%)

Maximum Value = 7942 Ω

Minimum Value = 6193 Ω

% Maximum Deviation > Average = 18%

% Maximum Deviation < Average = 8%

A similar study was conducted for variations in β_{ave} with the following results:

Beta:

Average = 3467 Kelvin

Standard Deviation = 96 K (2.8%)

Maximum Value = 3620 K

Minimum Value = 3119 K

% Maximum Deviation > Average = 4.4%

% Maximum Deviation < Average = 10%

The minimum value is an outlier, when it is removed we obtain:

Average = 3472 Kelvin

Standard Deviation = 85 K (2.4%)

Maximum Value = 3620 K

Minimum Value = 3251 K

% Maximum Deviation > Average = 4.4%

% Maximum Deviation < Average = 6.2%

The deviations--both greater and lesser than average--are outside the manufacturer's stated 3% tolerance. Tolerance variations for both R_{To} and β will have little consequence if each sensor is allowed to retain its own unique constants in the linear approximation, other than the fact that β will be linear over a different temperature range.

In the experimental determination of the dissipation constant it has been noticed that approximately 1 of 8 thermistor/sensors gives a dissipation constant, which is unusually lower or higher than expected. For the 64 sensors calibrated to date, the following information was obtained:

Average $\delta = 2.868\text{x}10^{-4}$ W/°C

Standard Deviation = $0.534\text{x}10^{-4}$ W/°C

Predicted Uncertainty = $0.23\text{x}10^{-4}$ W/°C

Though it is expected that there will be variations in δ due to varying heat-transfer mechanisms, these larger-than-expected fluctuations are altering the self-heating corrective term such that the corrected temperatures are as much as 0.2 - 0.3°C in error with the rest of the sensors in the batch. When this occurs, the sensor is often recalibrated, usually with the same result. The exact cause of this is not known, but a nonlinear power/conductance relationship in the conductive epoxy used to electrically connect the thermistors leads to the terminals has been considered. No literature has been found to support this hypothesis, however. Thermistor instability is another possibility, but neither has this been confirmed.

The current method of dealing with this discrepancy is by implementing an average dissipation-constant calculated from existing data and given above as $2.867\text{x}10^{-4}$ W/°C. Program DC_BETA.BAS was altered to use this average δ value. The new program, AVG_DCB.BAS, does not iterate on δ and β, but rather calculates thermistor constants using the average δ value. Figure 15 shows absolute-error curves for Batch-19 for which the average δ value was implemented. Note that the error curves are all tied down at about 24°C, the temperature at which the individual unique dissipation constants were calculated for this batch.

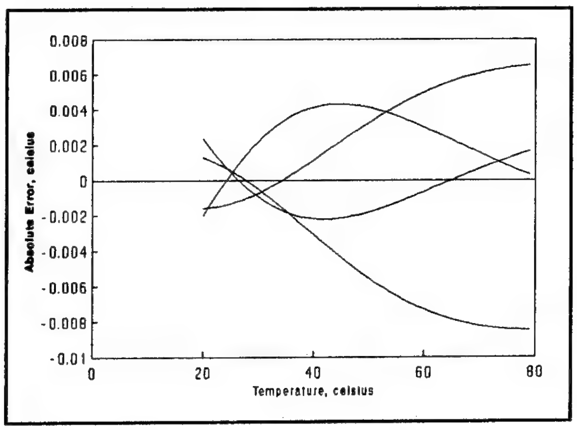

Fig. 15. Absolute-error curves for Batch-11 for which an average δ was used.

The high precision implied by this error plot is supportive of the use of an average δ. Additional studies may have to be taken along these lines in order to prove the validity of this approach.

CONCLUSIONS AND RECOMMENDATIONS

A procedure has been developed and utilized for the calibration of thermistors mounted in simulated skin sensors. An uncertainty analysis has been performed based on a linear approximation of the temperature/resistance characteristics of negative-temperature-coefficient thermistors. Expressions were derived, and uncertainties predicted for both absolute- and relative-temperature measurements.

In the analysis, a number of parametric measurement schemes were considered. The accuracy of temperature measurements made with a NBS calibrated thermometer was compared to those made with a precalibrated thermistor with the result that thermometer measurements are found to be the more accurate. Resistance values obtained through voltage measurements were found to have a lesser degree of uncertainty than those made with an available ohmmeter. Through the experimental determination of β and R_{T0}, the analysis

predicts a temperature uncertainty of 0.1-0.2°C over the temperature range of 25 - 50°C.

In addition, a measurement scheme was devised for the experimental determination of thermistor self-heating due to power dissipation, and temperature correction terms have been implemented for this and thermometer emergent-stem errors.

Software was written which calculates thermistor constants using voltage and temperature measurements while simultaneously predicting uncertainties in the calculated values, so that the integrity of each calibration run can be closely monitored.

Results from the 64 sensors calibrated as thermistors to date show variations of less than 0.1°C; within the predicted temperature-measurement uncertainty and required ±0.5°C accuracy.

A decision will have to be made concerning the form of the expression used to relate temperature to resistance in the ATRES/DAAS data processing software. The most precise temperature/resistance relationship is obtained through the polynomial expression (Eq. 7). The constants a_0, a_1, and a_3 are not currently being determined for each thermistor/sensor. The software is, however, setup for this purpose and it would not require much time to complete the task. In this case, the self-heating correction term must still be utilized in order to account for variations in source voltage, and hence, variations in dissipated power.

The error (±0.1°C) inherent in the linear approximation due to nonlinear β is almost negligible when one considers that our temperature-measurement uncertainty is between 0.1 and 0.2°C. For this reason, it is believed that the linear approximation represents an adequate relationship between temperature and resistance for our purposes.

If processing speed is a consideration (real time display), the linear approximation can be used with average β, R_{T0}, and δ.

In the event that greater precision is required for a particular application, a higher degree of accuracy can be obtained with a more precise ohmmeter for resistance measurements and a factory-calibrated thermistor for

temperature measurements. Essentially all of the software developed for this calibration procedure is currently applicable, or can be altered to accommodate a more precise calibration method.

It is believed that the calibration process presented here represents a reliable technique by which to achieve the required temperature-measurement precision of the new ATRES/DAAS system currently being developed at Natick.

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APPENDIX COMPUTER PROGRAMS

1. Program ERROR.BAS
2. Program RTNOT.BAS
3. Program RTNOTA.BAS
4. Program DC_BETA.BAS

1. Program ERROR.BAS

```
5   REM Program ERROR.BAS  Brian Kimball, 2/19/92
10  REM Calculates uncertainty in absolute and delta T measurements
20  INPUT "BETA, KELVIN B=", B
30  INPUT "Reference temperature, celsius Tr=", Tr
40  INPUT "Initial temperature, celsius Ti=", Ti
50  INPUT "Later temperature, celsius Tn=", Tn
60  REM INPUT "Resistor, ohms R=", R
62  R = 9998.8
65  INPUT "Thermistor resistance at reference temperature, ohms RTr=", RTr
70  INPUT "Source voltage, volts es=", es
80  INPUT "Uncertainty in Beta, Kelvin WB=", WB
90  INPUT "Uncertainty in reference temperature resistance, ohms WRTr=", WRTr
100 REM INPUT "Uncertainty in resistor resistance, ohms WR=", WR
105 WR = .2
120 INPUT "Uncertainty in reference temperature, celsius WTr=", WTr
125 REM
130 Tr = Tr + 273.15
135 Ti = Ti + 273.15
140 Tn = Tn + 273.15
150 RTi = RTr * EXP(B * ((1 / Ti) - (1 / Tr)))
155 Wes = es * .0001
160 eri = R * es / (RTi + R)
170 Weri = eri * .0001
180 RTn = RTr * EXP(B * ((1 / Tn) - (1 / Tr)))
190 ern = R * es / (RTn + R)
200 Wern = ern * .0001
205 REM
210 REM PARTIALS TO FIND ERROR IN RTi AND RTn
220 P1i = ((es / eri) - 1)
230 P2i = R / eri
240 P3i = R * es / (eri ^ 2)
250 WRTi = ((P1i * WR) ^ 2 + (P2i * Wes) ^ 2 + (P3i * Weri) ^ 2) ^ .5
255 REM PRINT "WRTi=", WRTi
260 P1n = ((es / ern) - 1)
270 P2n = R / ern
280 P3n = R * es / (ern ^ 2)
290 WRTn = ((P1n * WR) ^ 2 + (P2n * Wes) ^ 2 + (P3n * Wern) ^ 2) ^ .5
295 REM
299 REM
300 REM PARTIALS TO FIND ABSOLUTE ERROR IN Ti AND Tn
310 P4i = (B * Tr ^ 2) / (RTi * (Tr * LOG(RTi / RTr) + B) ^ 2)
320 P5i = (B * Tr ^ 2) / (RTr * (Tr * LOG(RTi / RTr) + B) ^ 2)
```

```
330 P6i = (Tr ^ 2) * LOG(RTi / RTr) / ((Tr * LOG(RTi / RTr) + B)
    ^ 2)
335 P7i = B ^ 2 / ((Tr * LOG(RTi / RTr) + B) ^ 2)
340 WTi = ((P4i * WRTi) ^ 2 + (P5i * WRTr) ^ 2 + (P6i * WB) ^ 2 +
    (P7i * WTr) ^ 2) ^ .5
350 REM
360 P4n = (B * Tr ^ 2) / (RTn * (Tr * LOG(RTn / RTr) + B) ^ 2)
370 P5n = (B * Tr ^ 2) / (RTr * (Tr * LOG(RTn / RTr) + B) ^ 2)
380 P6n = (Tr ^ 2) * LOG(RTn / RTr) / ((Tr * LOG(RTn / RTr) + B)
    ^ 2)
385 P7n = B ^ 2 / ((Tr * LOG(RTn / RTr) + B) ^ 2)
390 WTn = ((P4n * WRTn) ^ 2 + (P5n * WRTr) ^ 2 + (P6n * WB) ^ 2 +
    (P7n * WTr) ^ 2) ^ .5
400 REM
410 REM PARTIALS TO CALCULATE UNCERTAINTY IN DELTA T
420 DEN = (Tr * LOG(RTn / RTr) + B) * (Tr * LOG(RTi / RTr) + B)
430 P7 = (((DEN) * (Tr ^ 2 * LOG(RTi / RTn))) - ((B * Tr ^ 2 *
    LOG(RTi / RTn)) * (2 * B + Tr * (LOG((RTi * RTn) / (R * Tr ^
    2)))))) / (DEN ^ 2)
440 P8 = (((DEN) * (2 * B * LOG(RTi / RTn))) - ((B * Tr ^ 2 *
    LOG(RTi / RTn)) * (2 * Tr * LOG(RTn / RTr) * LOG(RTi / RTr) +
    B * LOG((RTi * RTn) / (RTr ^ 2))))) / (DEN ^ 2)
450 P9 = -((B * Tr ^ 2 * LOG(RTi / RTn)) * ((Tr / RTr) * (2 * B +
    Tr * LOG((RTn * RTi) / (RTr ^ 2))))) / (DEN ^ 2)
460 P10 = ((DEN) * (B * (Tr ^ 2) / RTi) - ((B * (Tr ^ 3) / RTi) *
    LOG(RTi / RTn)) * (Tr * LOG(RTn / RTr) + B)) / (DEN ^ 2)
470 P11 = ((DEN) * (-B * (Tr ^ 2) / RTn) - ((B * Tr ^ 3 * LOG(RTi
    / RTn) / RTn) * (Tr * LOG(RTi / RTn) + B))) / (DEN ^ 2)
480 WDelT = ((P7 * WB) ^ 2 + (P8 * WTr) ^ 2 + (P9 * WRTr) ^ 2 +
    (P10 * WRTi) ^ 2 + (P11 * WRTn) ^ 2) ^ .5
485 PRINT
490 REM
500 REM PRINT RESULTS
510 PRINT "The uncertainty in the absolute initial temp Ti is",
    WTi
520 PRINT "The uncertainty in the absolute later temp Tn is", WTn
530 PRINT "The uncertainty in the Delta-T measurement is", WDelT
535 PRINT
540 PRINT "The uncertainty in Delta-T due to the uncertainty in:"
550 WP7 = P7 * WB
560 PRINT "  Beta", WP7
570 WP8 = P8 * WTr
580 PRINT "  Tr", WP8
590 WP9 = P9 * WRTr
600 PRINT "  RTr", WP9
610 WP10 = P10 * WRTi
620 PRINT "  RTi", WP10
630 WP11 = P11 * WRTn
640 PRINT "  RTn", WP11
650 PRINT
660 PRINT "Uncertainty in Ti due to uncertainty in:"
670 PRINT " Beta", (P6i * WB)
```

```
680 PRINT " Tr", (P7i * WTr)
690 PRINT " RTr", (P5i * WRTr)
700 PRINT " RTi", (P4i * WRTi)
710 PRINT
720 PRINT "Uncertainty in Tn due to uncertainty in:"
730 PRINT " Beta", (P6n * WB)
740 PRINT " Tr", (P7n * WTr)
750 PRINT " RTr", (P5n * WRTr)
760 PRINT " RTn", (P4n * WRTn)
770 PRINT
800 END
```

2. Program RTNOT.BAS

```
10  REM program RTNOT.BAS, Brian Kimball, 2/20/92
15  REM This program computes values of RTo.  It also
20  REM calculates uncertainty in experimentally determined
30  REM values of RTo. This version used for calibrating
31  REM reference thermistors.
40  INPUT "Ambient temperature, celsius T=", T
41  IF T = 0 THEN
42  GOTO 210
43  ELSE
44  GOTO 50
45  END IF
50  REM INPUT "Uncertainty in ambient temperature, celsius WT=",
    WT
55  WT = .05
60   INPUT "Thermistor resistance at ambient temperature, ohms
    RT=", RT
70  REM INPUT "Uncertainty in RT, ohms WRT=", WRT
75  WRT = 3
80  T = T + 273.15
90  B = 3474
100 WB = 18
110 Tr = 298.15
111 RTr = RT / (EXP(B * (1 / T - 1 / Tr)))
120 REM  Calculate partials
130 P1 = 1 / (EXP(B * (1 / T - 1 / Tr)))
140 P2 = RT * (1 / T - 1 / Tr) / (EXP(B * (1 / T - 1 / Tr)))
150 P3 = RT * B / ((T ^ 2) * (EXP(B * (1 / T - 1 / Tr))))
160 WRTr = ((P1 * WRT) ^ 2 + (P2 * WB) ^ 2 + (P3 * WT) ^ 2) ^ .5
165 PE1 = ((RTr - 10000) / 10000) * 100
167 PE2 = (WRTr / RTr) * 100
169 PRINT
170 PRINT "Reference temp resistance is", RTr
180 PRINT "Thermistor tolerance is, percent", PE1
190 PRINT "The uncertainty in this measurement (RTr) is, ohms",
    WRTr
200 PRINT "or, percent", PE2
201 PRINT "Resistance term"; (P1 * WRT)
202 PRINT "Beta term"; (P2 * WB)
203 PRINT "Temperature term"; (P3 * WT)
204 PRINT
205 PRINT
206 GOTO 40
210 END
```

3. Program RTNOTA.BAS

```
10  REM program RTNOTA.bas, Brian Kimball, 2/24/92
20  REM This program calculates the reference temperature
30  REM resistance of a thermistor.  It calculates the ambient
31  REM temperature by using the resistance value of a
32  REM pre-calibrated reference thermistor (T51).It uses this
33  REM temperature and resistance value of the test thermistor
34  REM (input) at this ambient temperature to determine the
35  REM reference temperature resistance of the test thermistor.
36  REM The reference temperature used here is 25 degrees C.

40  INPUT "Reference thermistor resistance, ohms RT51=", RT51
41  IF RT51 = 0 THEN
42  GOTO 210
43  ELSE
44  GOTO 46
45  END IF

46  INPUT "Test thermistor resistance, ohms RT=", RT
55  WT = .25
75  WRT = 20
80  B = 3474
85  WB = 104
90  TR = 298.15
91  RT51r = 10316

92  T = B * TR / (TR * LOG(RT51 / RT51r) + B)
115 RTr = RT / (EXP(B * (1 / T - 1 / TR)))

120 REM  Calculate partials
130 P1 = 1 / (EXP(B * (1 / T - 1 / TR)))
140 P2 = RT * (1 / T - 1 / TR) / (EXP(B * (1 / T - 1 / TR)))
150 P3 = RT * B / ((T ^ 2) * (EXP(B * (1 / T - 1 / TR))))
160 WRTr = ((P1 * WRT) ^ 2 + (P2 * WB) ^ 2 + (P3 * WT) ^ 2) ^ .5

165 PE1 = ((RTr - 10000) / 10000) * 100
167 PE2 = (WRTr / RTr) * 100

168 PRINT
169 PRINT "Ambient temp, celsius", (T - 273.15)
170 PRINT "Reference temp resistance is", RTr
180 PRINT "Thermistor tolerance is, percent", PE1
190 PRINT "The uncertainty in this measurement (RTr) is, ohms",
          WRTr
200 PRINT "or, percent", PE2
201 PRINT "Resistance term"; (P1 * WRT)
202 PRINT "Beta term"; (P2 * WB)
203 PRINT "Temperature term"; (P3 * WT)
204 PRINT
```

```
205 PRINT
206 GOTO 40
210 END
```

4. Program DC_BETA.BAS

```
420   REM Program DC_BETA.BAS, Brian Kimball, 3/3/92
500   REM
510   REM This program calculates thermistor constants from
520   REM temperature/voltage data.
740   REM
760   REM        DEFINE VARIABLES
780   REM R - voltage divider resistor resistance
800   REM TL - low temperature (thermometer room temperature)
820   REM VSL - source voltage at low temperature
840   REM VTL - voltage across resistor R at low temperature
860   REM VSLP - source voltage at low temperature and low power
880   REM VTLP - voltage across resistor R at low temperature and
      low power
900   REM RTL - resistance across thermistor at low temperature
920   REM ETL - voltage across thermistor at low temperature
940   REM RTLP - resistance across thermistor at low temperature
      and low power
960   REM ETLP - voltage across thermistor at low temperature and
      low power
980   REM TM - mid temperature (thermometer ~36 C)
1000  REM VSM - source voltage at mid temperature
1020  REM VTM - voltage across resistor R at mid temperature
1040  REM TAM - thermometer stem temperature at mid temperature
1060  REM RTM - thermistor resistance at mid temperature
1080  REM ETM - thermistor voltage at mid temperature
1100  REM TH - high temperature (thermometer ~50 C)
1120  REM VSH - source voltage at high temperature
1140  REM VTH - voltage across resistor R at high temperature
1160  REM TAH - thermometer stem temperature  at high temperature
1180  REM RTH - thermistor resistance at high temperature
1200  REM ETH - thermistor voltage at high temperature
1220  REM PL - power dissipated in thermistor at low temperature
1240  REM PLP - power dissipated in thermistor at low temperature
      low power
1260  REM PM - power dissipated in thermistor at mid temperature
1280  REM PH - power dissipated in thermistor at high temperature
1300  REM DCI - assumed value of dissipation constant (temporary
      variable)
1320  REM DC - dissipation constant
1340  REM WRT - uncertainty in thermistor resistance
1360  REM WRTR - uncertainty in reference temperature thermistor
      resistance
1380  REM WT - uncertainty in temperature
1400  REM WTr - uncertainty in reference temperature
1420  REM T - temperature (temporary variable)
1440  REM Tr - reference temperature (temporary variable)
1460  REM RT - thermistor resistance (temporary variable)
```

```
1480   REM RTr - thermistor reference temperature resistance
       (temporary variable)
1500   REM BLM - beta between low and mid temperatures
1520   REM P1, P2, P3, P4 - partials for calculating uncertainty
       (temporary variables)
1540   REM WBLM - uncertainty in BLM
1560   REM BMH - beta between mid and high temperatures
1580   REM WBMH - uncertainty in BMH
1600   REM BLH - beta between low and high temperatures
1620   REM WBLH - uncertainty in BLH
1640   REM BAVE - average of betas
1660   REM EXTL - experimental low temperature
1680   REM EXTLP - experimental temperature at low power
1700   REM ERRL - absolute error in experimental low temperature
1720   REM EXTH - experimental high temperature
1740   REM ERRH - absolute error in experimental high temperature
1760   REM TLC - corrected actual thermistor low temperature
1780   REM TMC - corrected actual thermistor mid temperature
1800   REM THC - corrected actual thermistor high temperature
1820   REM RTref - final reference temperature resistance at 36
       degrees C
1840   REM
1860   R = 9998.8
1862   PRINT "Certain variables within a calibration run do not
       change; it"
1864   PRINT "is only necessary to enter them once per batch.  If
       you are "
1866   PRINT "beginning a new batch answer 'Y' at the prompt.
       Enter 'N' if"
1868   PRINT "you have already defined 'batch' variables."
1870   g$ = "n"
1872   PRINT
1874   PRINT "Are you beginning a new calibration batch?"

1875   DO UNTIL g$ = "Y" OR g$ = "N"
1876      INPUT "Enter Y/N: ", g$
1877   LOOP

1878   IF g$ = "Y" THEN
1879         PRINT "YES"
1880         GOTO 1885
1881      ELSE
1882         PRINT "NO"
1883         GOTO 2080
1884   END IF

1885   PRINT "Input 'batch' variables"
1890   INPUT " TL = ", TL
1900   INPUT "VSL = ", VSL
1920   INPUT "LOW POWER VSL = ", VSLP
1940   INPUT " TM = ", TM
1960   INPUT "VSM = ", VSM
```

```
2000  INPUT "STEM TEMP (M) = ", TAM
2020  INPUT " TH = ", TH
2040  INPUT "VSH = ", VSH
2060  INPUT "STEM TEMP (H) = ", TAH

2080  INPUT "VTL = ", VTL
2100  INPUT "LOW POWER VTL=", VTLP
2120  RTL = R * ((VSL / VTL) - 1)

2140  ETL = (VSL * RTL) / (RTL + R)
2160  RTLP = R * ((VSLP / VTLP) - 1)
2180  ETLP = (VSLP * RTLP) / (RTLP + R)
2200  REM
2210  INPUT "VTM = ", VTM
2220  RTM = R * ((VSM / VTM) - 1)
2240  ETM = (VSM * RTM) / (RTM + R)
2260  REM
2280  INPUT "VTH = ", VTH
2300  RTH = R * ((VSH / VTH) - 1)
2320  ETH = (VSH * RTH) / (RTH + R)
2340  REM
2360  REM CORRECTION FOR EMERGENT STEM
2380  TMC = TM + (.00016 * 111 * (TM - TAM))
2400  THC = TH + (.00016 * 111 * (TH - TAH))
2420  REM CORRECTION FOR THERMISTOR SELF-HEATING
2440  REM CALCULATE POWER DISSIPATED
2460  PL = ((ETL ^ 2) / RTL)
2480  PLP = ((ETLP ^ 2) / RTLP)
2500  PM = ((ETM ^ 2) / RTM)
2520  PH = ((ETH ^ 2) / RTH)
2540  TLC = TL + 273.15
2560  TMC = TMC + 273.15
2580  THC = THC + 273.15
2600  REM DISSIPATION CONSTANT
2620  DCI = .0003
2640  DC = .00033
2650  PRINT "Iterating on dissipation constant..."
2660  IF ABS(DC - DCI) < (.000001) THEN
2680     GOTO 4120
2700  ELSE
2720     GOTO 2760
2740  END IF
2760     DC = (DC + DCI) / 2
2780     REM ADD SELF HEATING TERM
2800     TLC = TLC + (PL / DC)
2820     PRINT DC
2840     TMC = TMC + (PM / DC)
2860     THC = THC + (PH / DC)
2880     REM
2900     WRT = 3
2920     WRTr = 3
2940     WT = .05
```

```
2960      WTr = .05
2980      REM
3000      REM CALCULATE BETA-LM AND BETA UNCERTAINTY
3100      T = TLC
3120      Tr = TMC
3140      RT = RTL
3160      RTr = RTM
3180      BLM = (T * Tr / (Tr - T)) * (LOG(RT / RTr))
3200      REM
3220      REM CALCULATE PARTIALS FOR BETA-LM
3240      P1 = T * Tr / (RT * (Tr - T))
3260      P2 = T * Tr / (RTr * (Tr - T))
3280      P3 = ((Tr ^ 2) / ((Tr - T) ^ 2)) * LOG(RT / RTr)
3300      P4 = ((T ^ 2) / ((Tr - T) ^ 2)) * LOG(RT / RTr)
3320      WBLM = ((P1 * WRT) ^ 2 + (P2 * WRTr) ^ 2 + (P3 * WT) ^ 2
           + (P4 * WTr) ^ 2) ^ .5
3340      REM
3360      REM CALCULATE BETA-MH AND BETA UNCERTAINTY
3380      T = THC
3400      Tr = TMC
3420      RT = RTH
3440      RTr = RTM
3460      BMH = (T * Tr / (Tr - T)) * (LOG(RT / RTr))
3480      REM
3500      REM CALCULATE PARTIALS FOR BETA-MH
3520      P1 = T * Tr / (RT * (Tr - T))
3540      P2 = T * Tr / (RTr * (Tr - T))
3560      P3 = ((Tr ^ 2) / ((Tr - T) ^ 2)) * LOG(RT / RTr)
3580      P4 = ((T ^ 2) / ((Tr - T) ^ 2)) * LOG(RT / RTr)
3600      WBMH = ((P1 * WRT) ^ 2 + (P2 * WRTr) ^ 2 + (P3 * WT) ^ 2
          + (P4 * WTr) ^ 2) ^ .5
3620      REM
3640      REM CALCULATE BETA-LH AND BETA UNCERTAINTY
3660      T = TLC
3680      Tr = THC
3700      RT = RTL
3720      RTr = RTH
3740      BLH = (T * Tr / (Tr - T)) * (LOG(RT / RTr))
3760      REM
3780      REM CALCULATE PARTIALS FOR BETA-LH
3800      P1 = T * Tr / (RT * (Tr - T))
3820      P2 = T * Tr / (RTr * (Tr - T))
3840      P3 = ((Tr ^ 2) / ((Tr - T) ^ 2)) * LOG(RT / RTr)
3860      P4 = ((T ^ 2) / ((Tr - T) ^ 2)) * LOG(RT / RTr)
3880      REM
3900            REM Calculate temp at extremes using linear
          approximation.
3920      REM Also, calculate absolute error = T(actual)-T(calc)
3940      WBLH = ((P1 * WRT) ^ 2 + (P2 * WRTr) ^ 2 + (P3 * WT) ^ 2
          + (P4 * WTr) ^ 2) ^ .5
3960      BAVE = (BLM + BMH + BLH) / 3
3980        EXTL = (BLH * TMC / (TMC * LOG(RTL / RTM) + BLH)) -
```

```
         273.15
4000      EXTLP = (BLH * TMC / (TMC * LOG(RTLP / RTM) + BLH)) -
         273.15
4020     DCI = (PL - PLP) / (EXTL - EXTLP)
4040     TLC = TLC - (PL / DC)
4060     TMC = TMC - (PM / DC)
4080     THC = THC - (PH / DC)
4100  GOTO 2660

4120  DC = (DC + DCI) / 2
4140  TLC = TLC + (PL / DC)
4160  TMC = TMC + (PM / DC)
4180  THC = THC + (PH / DC)
4200  ERRL = TLC - EXTL - 273.15
4220  EXTH = (BLH * TMC / (TMC * LOG(RTH / RTM) + BLH)) - 273.15
4240  ERRH = THC - EXTH - 273.15
4260  TLCC = TLC - 273.15
4280  TMCC = TMC - 273.15
4300  THCC = THC - 273.15
4320  REM
4340    REM  Calculate  reference  temperature  resistance  at  36
      degrees,celsius.
4360  RTref = RTM / (EXP(BLH * ((1 / TMC) - (1 / 309.15))))
4380  Z1 = PL / DC
4400  Z2 = PM / DC
4420  Z3 = PH / DC
4440  REM
4460  PRINT
4480  PRINT "", "BETA, kelvin", "UNCERTAINTY", "RT, ohms"
4500  PRINT "BLM", BLM, WBLM, RTL
4520  PRINT "BMH", BMH, WBMH, RTM
4540  PRINT "BLH", BLH, WBLH, RTH
4560  PRINT "BAVE", BAVE
4580  PRINT
4600  PRINT "", "TEMP", "EXP TEMP", "ERROR"
4620  PRINT "LOW", TLCC, EXTL, ERRL
4640  PRINT "MED", TMCC, "", ""
4660  PRINT "HIGH", THCC, EXTH, ERRH
4680  PRINT
4700  PRINT "RT36 = ", RTref
4720  PRINT "DC = ", DC
4760  PRINT "SHL", "SHM", "SHH"
4780  PRINT Z1, Z2, Z3
4800  PRINT
4810  g$ = "n"
4820  PRINT "Do you wish to continue?"
4822  DO UNTIL g$ = "Y" OR g$ = "N"
4824     INPUT "Enter Y/N: ", g$
4826  LOOP

4827  IF g$ = "Y" THEN
4828        PRINT "YES"
```

```
4829          GOTO 1870
4830      ELSE
4831          PRINT "NO"
4832          GOTO 6000
4833  END IF
6000  END
```

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